

Experimental design for multi-level data: Improving our approach to power analysis using Monte Carlo simulation-based parameter recovery estimation

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What's the point of research?

My research question:

“Are ratings of comprehension predictive of assessed comprehension?”

““Will my study answer my research question?’ is the most fundamental question a researcher can ask when designing a study”

(Johnson et al., 2015, p. 133)

Adequately designing a study

Question:

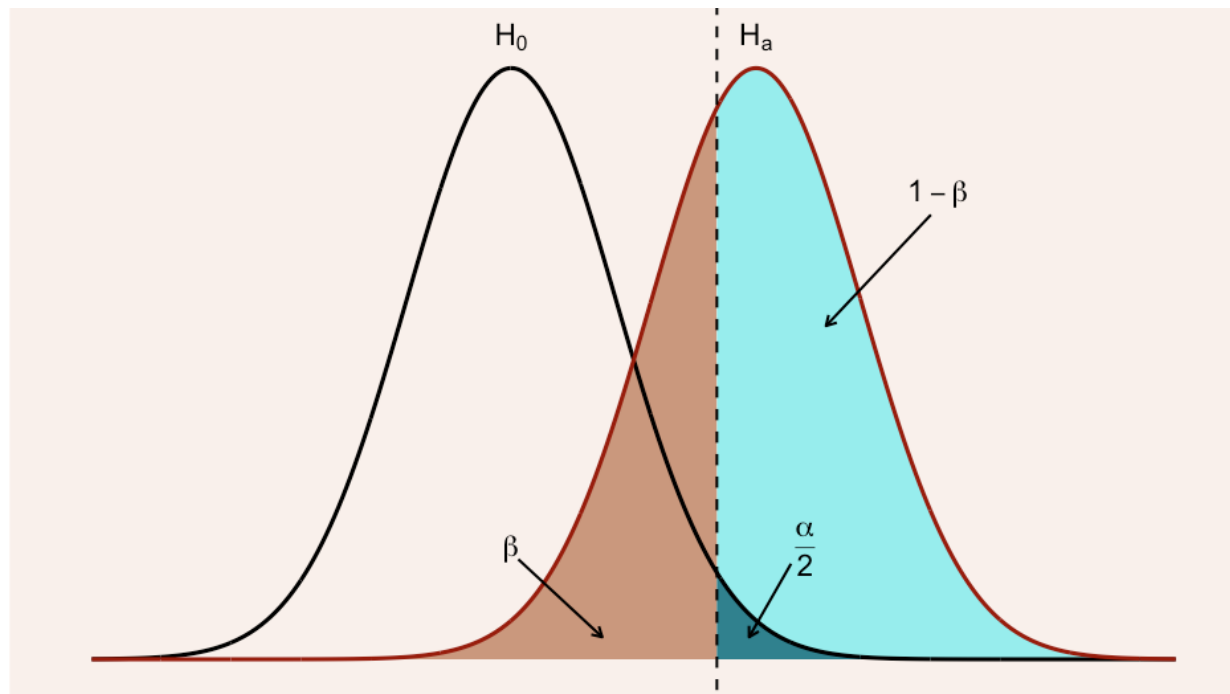
- Will my study be adequately powered to detect an effect of interest?

Answer:

- Do power analysis

What is power analysis?

Power = $P(\text{correctly reject } H_0)$



“Do power analysis”

- 1) Formulaic / analytic method
- 2) Simulation-based method

Formulaic / analytic approach

Test family	Statistical test			
t tests	Means: Difference between two independent means (two groups)			
Type of power	Software	Platform	URL ^a	Freely available?
A priori: Comp	Stand-alone programs			
Input Parameter	G*Power	Windows and macOS	http://www.gpower.hhu.de	Yes
Determine =>	PS	Windows	http://biostat.mc.vanderbilt.edu/wiki/Main/PowerSampleSize	Yes
	PASS	Windows	https://www.ncss.com/software/pass	No
	nQuery	Windows	https://www.statsols.com/nquery-sample-size-and-power-calculation-for-successful-clinical-trials	No
	R^b packages			
	pwr	Windows, macOS and Linux	https://cran.r-project.org/web/packages/pwr	Yes
	TrialSize	Windows, macOS and Linux	https://cran.r-project.org/web/packages/TrialSize	Yes
	PowerUpR ^c	Windows, macOS and Linux	https://cran.r-project.org/web/packages/PowerUpR	Yes
	powerSurvEpi	Windows, macOS and Linux	https://CRAN.R-project.org/package=powerSurvEpi	Yes
	SAS			
	PROC POWER	Windows and Linux	https://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/viewer.htm#power_toc.htm	No
	SPSS			
	SamplePower	Windows	https://www-01.ibm.com/marketing/iwm/iwm/docs/tnd/data/web/en_US/trialprograms/U741655136057W80.html	No
	Stata			
	power	Windows, macOS and Linux	https://www.stata.com/features/power-and-sample-size/	No
	Microsoft Excel			
	PowerUp ^c		http://www.causalevaluation.org/power-analysis.html	Yes ^d
	Specialist simulation software			
	IcebergSim	Windows	http://icebergsim.software.informer.com/versions/	Yes
	FACTS	Windows	https://www.berryconsultants.com/software/	No
	Clinical trial simulation	Windows and Linux	http://www.biopharmnet.com/innovation/trial_simulation/cts1.php	Yes ^e

(Hickey et al., 2018, Table 3)

Formulaic / analytic limitations

“advances [in specialist modelling techniques] have not been matched by the development of analytic formulae for sample size calculations under such models”

(Landau & Stahl, 2013, p. 325)

Off-the-shelf formula assumptions are rarely met

Bespoke closed-form equations can be designed, but can be difficult to define and inflexible

Simulation-based approach

Simulation-based power analyses can handle any design

Simulation-based power analyses can handle any data-generating mechanism

Separates the data-generating model from the analytic model
(Landau & Stahl, 2013)

In 'n' steps or less

1. Define the data-generating mechanism
2. Simulate many datasets
3. Perform an analysis on each dataset
4. Calculate performance

(Arnold et al., 2011; Johnson et al., 2015; Kontopantelis et al., 2016; Landau & Stahl, 2013)

1. Define the data-generating mechanism

- Outcome distribution
- Sources of variance
- Covariate distributions
- Effect distributions

Making assumptions of the generative model

What's sensible is defensible:

A plausible range of parameter values should, with careful consideration and transparent justification, be assumed based on knowledge of the topic and study design.

2-4. Simulation software options

SIMR (R; Green & MacLeod, 2016)

MLPowSim (MLwiN; Browne, Gollalizadeh, & Parker, 2009)

POWERSIM (Stata; Luedicke, 2013)

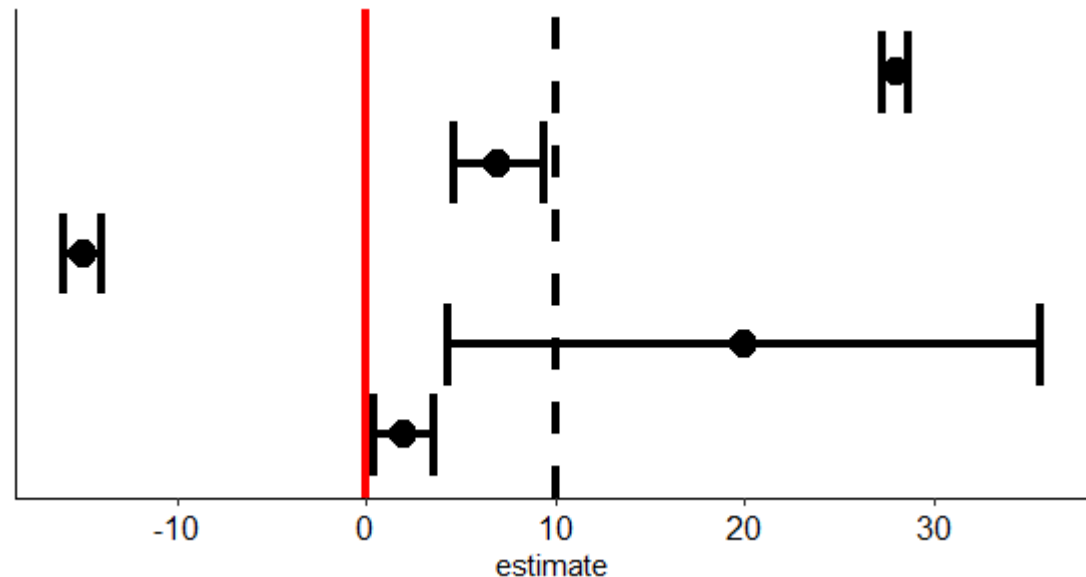
Idpower (Stata; Kontopantelis, 2018)

Power analysis is flawed

“a narrow emphasis on statistical significance is placed as the primary focus of study design”

(Gelman & Carlin, 2014, p. 641)

Example: $\beta = 10$



Power analysis can be broader

Conventional power (NHST) is one form of power, and power analysis can be thought of more broadly, in terms of different goals.


(Gelman & Carlin, 2014; Hickey et al., 2018; Johnson et al., 2015; Kruschke, 2014; Landau & Stahl; 2013)

Reframe the question

Question:

- ~~- Will my study be adequately powered to detect an effect of interest?~~
- Will my study be adequately designed to accurately recover an effect of interest?

Answer:

- ~~- Do power analysis~~
 - Do parameter recovery analysis
- 

“Do parameter recovery analysis”

1. Define the data-generating mechanism
2. Simulate many datasets
3. Perform an analysis on each dataset
4. Calculate performance¹

¹ in a more informative way

Defining parameter recovery

Two types of precision:

1. Estimate
2. Uncertainty

Parameter is recovered if:

The estimate is within a specified range and the associated uncertainty is within a specified range

Estimate precision

E.g. Estimate precision of β +/- 25%

Where β is the effect of interest

$$\beta = 10$$

$$7.5 \geq \hat{\beta} \leq 12.5$$

Frequentist error precision

E.g. Error precision of $SE_{\hat{\beta}} \leq 1.5$

Where $SE_{\hat{\beta}}$ is the estimated standard error associated with $\hat{\beta}$

$$95\% \text{ CI} = \hat{\beta} \pm SE_{\hat{\beta}} * 1.96$$

$$\hat{\beta} = 7.5, 95\% \text{ CI} = [4.56, 10.44]$$

$$\hat{\beta} = 10, 95\% \text{ CI} = [7.06, 12.94]$$

$$\hat{\beta} = 12.5, 95\% \text{ CI} = [9.56, 15.44]$$

Bayesian error precision

E.g. Error precision of $\hat{\beta} \pm 3$...

Contained within the credible intervals or posterior HDI

$$95\% \text{ CI}_{\hat{\beta}} = [\hat{\beta} - 3, \hat{\beta} + 3]$$

$$80\% \text{ HDI}_{\hat{\beta}} = [\hat{\beta} - 3, \hat{\beta} + 3]$$

Example: My study

1. Define the data-generating mechanism

$$Y_{ijkl} = \text{Bernoulli}(\Theta_{ijkl})$$

$$\Theta_{ijkl} = \beta_{0ijkl} + \beta_{1i}x_{1i} + \beta_{2i}x_{2i} + \beta_{3i}x_{3i} + \beta_{4i}x_{4i}$$

x_1 = Comprehension ability	i = participant
x_2 = Vocabulary	j = text
x_3 = Topic familiarity	k = question
x_4 = Rated comprehension	l = observation

$$\beta_{0ijkl} = \gamma_0 + u_{0il} + u_{0ij} + u_{0ik}$$

$$u_{0il} = N(u_{0i}, \sigma^2)$$

$$u_{0i} = N(0, \sigma^2)$$

...

$$\beta_{1i} = N(\mu, \sigma^2)$$

$$\beta_{2i} = N(\mu, \sigma^2)$$

...

Example: My study

2. Simulate many datasets

Texts: 5, 10, 15

Participants: 50-500

Example: My study

3. Perform an analysis on the datasets

```
clmm(count ~ (1 | participant) + (1 | text) + comprehension.ability +  
vocabulary.score + topic.familiarity + rated.comprehension)
```

Example: My study

4. Calculate performance

Estimate precision: 50% of β

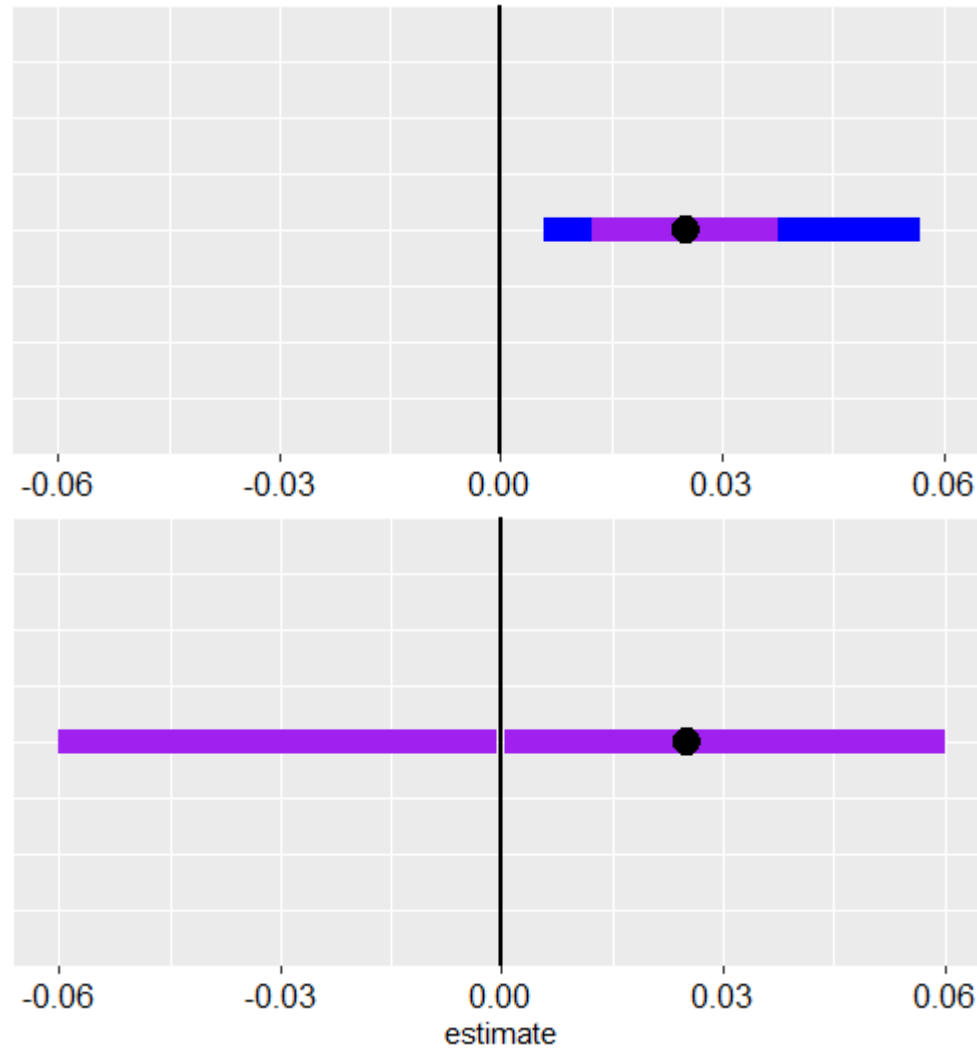
$$0.5\beta \geq \hat{\beta} \leq 1.5\beta$$

Error precision: 50% of $\hat{\beta}$

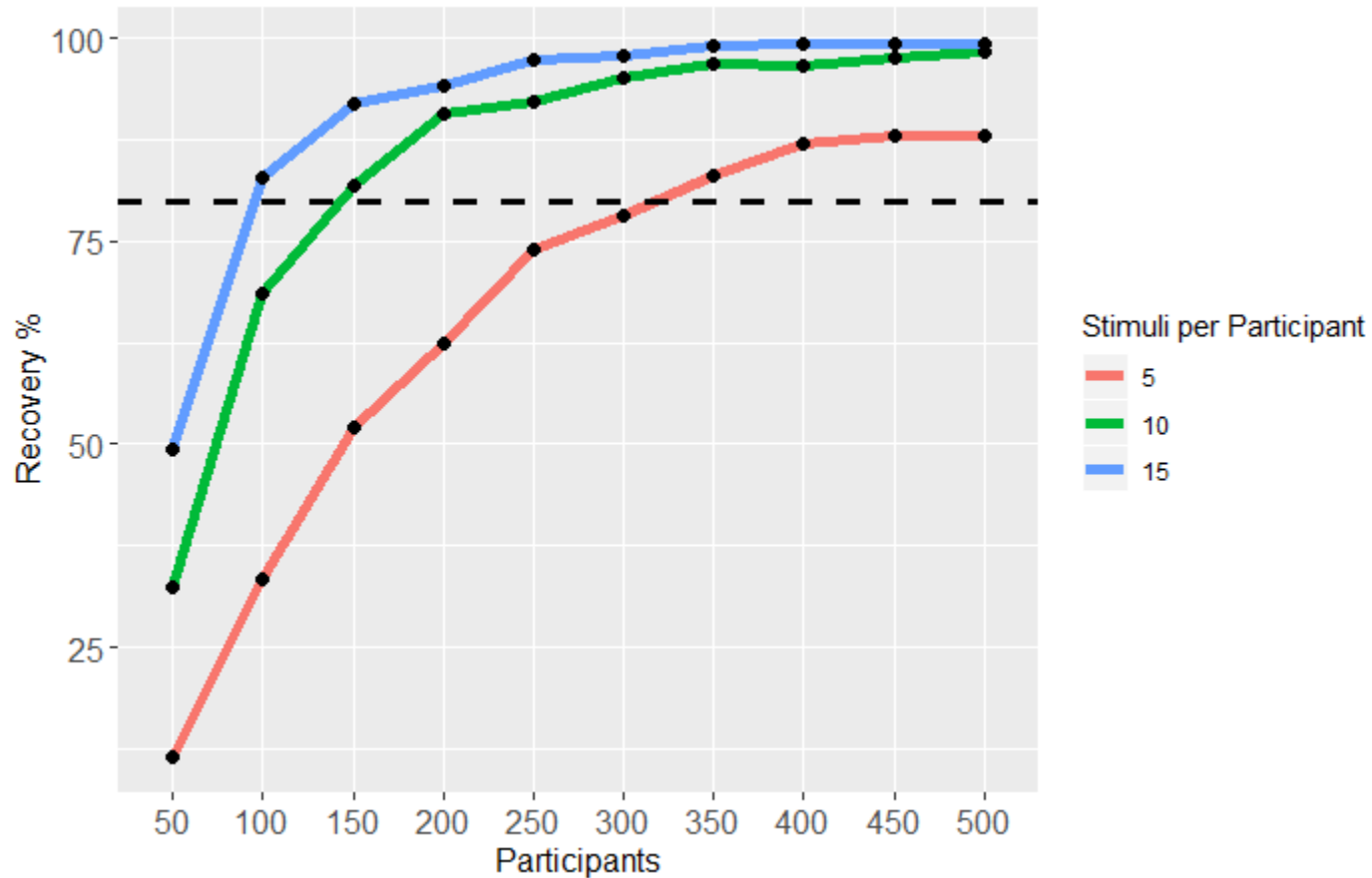
$$95\% \text{ UCI}_{\hat{\beta}} \geq 0.5 * 0.5\beta \text{ and } 95\% \text{ LCI}_{\hat{\beta}} \leq 0.5 * 1.5\beta$$

Example: My study

$\beta_{4\mu}$
0.025



Example: Result



Caution

Assumptions on parameters

Choosing parameter estimates is difficult

Time

Convergence



Available code

R package on GitHub – chaddlewick/spr (under development)

```
observedvariables = as.list(c(participant = "rep(1:20, each = 40)",
  qriscore = "rnorm(participant, 10, 2)",
  hlvascore = "rnorm(participant, 8, 0.5)",
  texts = "rep(1:10, times = 20, each = 4)",
  question = "rep(1:800)"))
effectvariables = as.list(c(intercept = "0.15",
  bparticipant = "rnorm(participant, mean=0, sd=0.4)",
  bqriscore = "rnorm(participant, 0.025, 0.001)",
  bhlvascore = "rnorm(participant, 0.02, 0.001)",
  btexts = "rnorm(texts, 0, 0.02)",
  bquestion = "rnorm(question, 0, 0.015)"))
outcomegeneration = as.list(c(outcome= "rbinom(observation, 1, dataset$py)",
  py = "dataset$intercept + dataset$bparticipant + dataset$bqriscore*dataset$qriscore +
  dataset$bhlvascore*dataset$hlvascore + dataset$btexts + dataset$bquestion"))
analyticmodel = "brm(outcome ~ (1|participant) + (1|texts) + qriscore + hlvascore, data=dataset, family =
  bernoulli(), cores = 2)"
```

References & Resources

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Thank you

Do you have any questions or feedback?

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