Dynamic Structural Equation Modeling of Intensive Longitudinal Data

Workshop for the Multilevel Conference Utrecht

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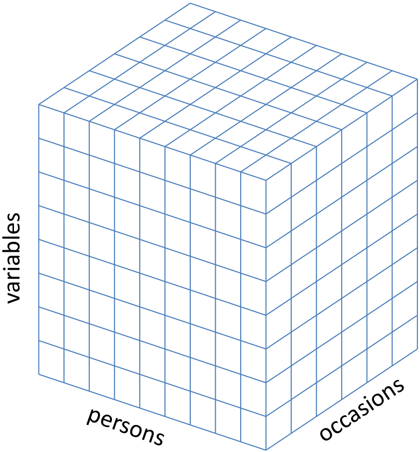
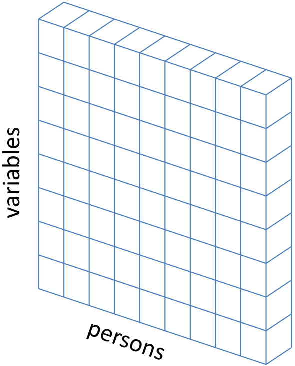
In collaboration with Bengt Muthén and Tihomir Asparouhov

Cattell's data box



Cross-sectional research: N is large, T=1



Cross-sectional research: A single snapshot

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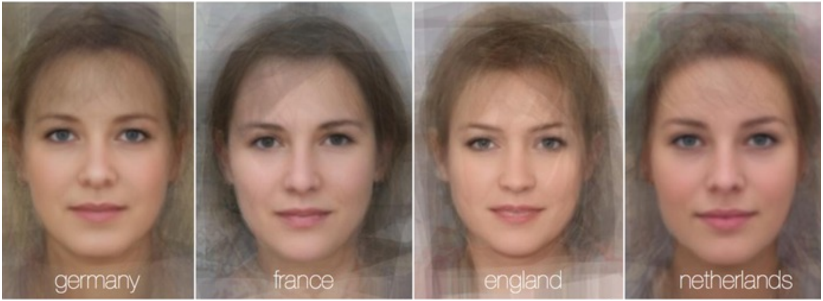
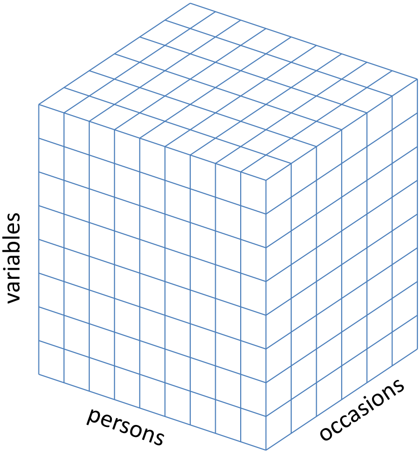
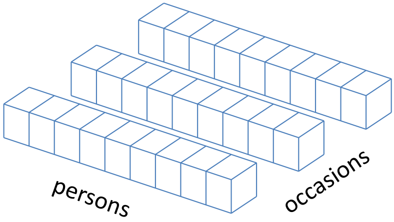


Cattell's data box

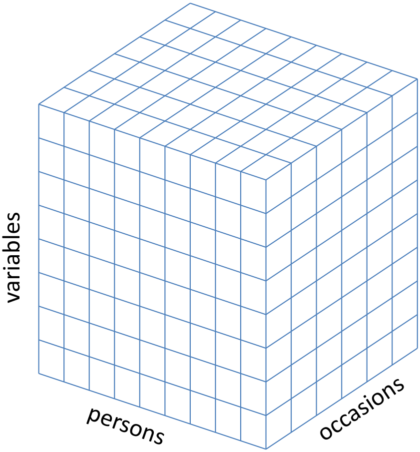


Panel research: N is large, T is small

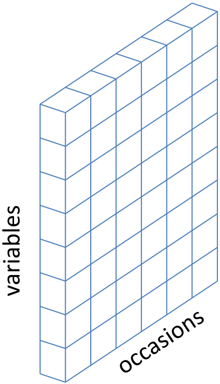


Panel research: A few snapshots

Cattell's data box

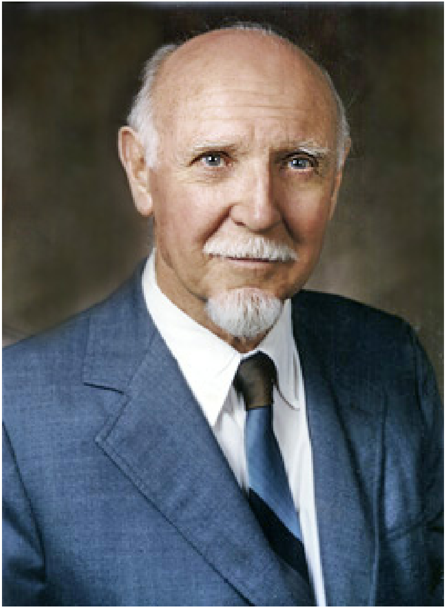


Time series data: N=1 and T is large



Time series analysis: Many snapshots

Pioneers of idiographic research in psychology



Idiographic (N=1) research in psychology

N=1 research has included:

- Cattell's P-technique: factor analysis of N=1 data
- Dynamic factor analysis: considering lagged relationships
- Measurement burst design: multiple waves of intensive measurements
- Intervention research: ABAB design etc.

Critique of this kind of research:

- within-person fluctuations are just **noise**
- results are **not generalizable**
- no one has these data

New technology

Smart phones

Smart glasses

Secure continuous remote
alcohol monitor (SCRAM)

Smart watches

Activity trackers

Intensive longitudinal data

Different forms of intensive longitudinal data:

- daily diary (DD); self-report end-of-day
- experience sampling method (ESM); self-report of subjective experience
- ecological momentary assessment (EMA); healthcare related self-report
- ambulatory assessment (AA); physiological measurements
- event-based measurements; self-report after a particular event
- observational measurements; expert rater

For more info on **methodology**, check out:

- Seminar of Tamlin Conner and Joshua Smyth on YouTube (https://www.youtube.com/watch?v=nQBBVp9vBIQ)
- Society for Ambulatory Assessment (http://www.saa2009.org/)
- Life Data (https://www.lifedatacorp.com/)
- Quantified Self (http://quantifiedself.com/)

Characteristics of these kind of data

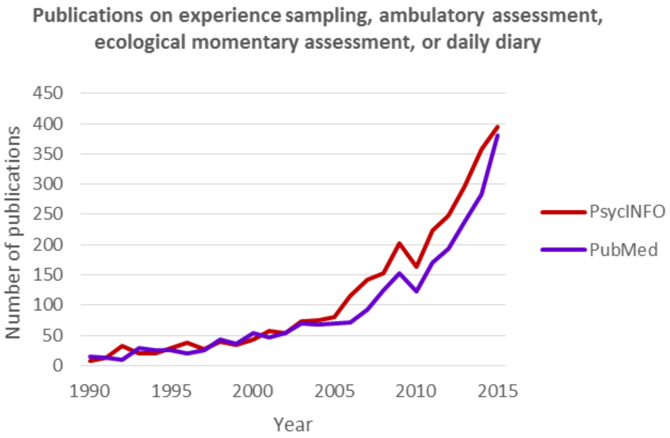
Data structure:

- one or more measurements per day
- typically for multiple days
- sometimes multiple waves (i.e., Nesselroade's measurement-burst design)

Advantages of ESM, EMA and AA

- no recall bias
- high ecological validity
- physiological measures over a large time span
- monitoring of symptoms and behavior, with new possibilities for feedback and intervention (e-Health and m-Health)
- window into the dynamics of processes

A paradigm shift



Taken from Hamaker and Wichers (2017)

Outline

- **Modeling the dynamics of ILD**
- Separating between-person and within-person variance
- Application 1: Daily negative affect and depressive symptomatology
- Application 2: Intervention study with ESM
- Application 3: Dyadic daily diary data
- Application 4: Latent AR(1) model
- Discussion

What is time series analysis?

Time series analysis is a class of techniques that is used in econometrics, seismology, meteorology, control engineering, and signal processing.

Main characteristics:

- N=1 technique
- T is large (say >50)
- concerned with *trends*, *cycles* and *autocorrelation structure* (i.e., serial dependency)
- goal: forecasting ($\neq$ prediction)

Lags

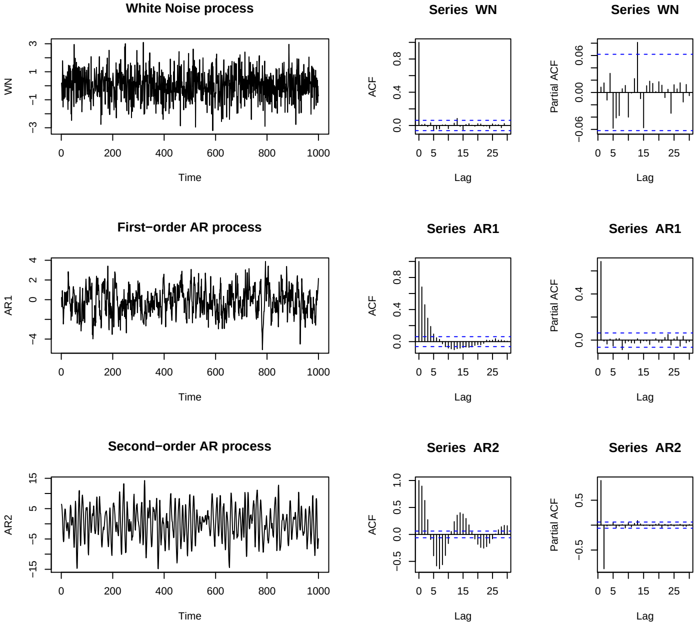
Y	Y at lag 1	Y at lag 2
y_1		
y_2	y_1	
y_3	y_2	y_1
y_4	y_3	y_2
y_5	y_4	y_3
y_6	y_5	y_4
y_7	y_6	y_5
y_8	y_7	y_6
$\dots$	$\dots$	$\dots$
y_T	y_{T-1}	y_{T-2}
	y_T	y_{T-1}
		y_T

Partial autocorrelation function (PACF)

Partial autocorrelation at lag k: The correlation between y_t and y_{t-k} after **removing the effect of the intermediate observations** (i.e., y_{t-1} to y_{t-k+1}).

Y	Y at lag 1	Y at lag 2
y_1		
y_2	y_1	
y_3	y_2	y_1
y_4	y_3	y_2
y_5	y_4	y_3
$\dots$	$\dots$	$\dots$
y_T	y_{T-1}	y_{T-2}
	y_T	y_{T-1}
		y_T

Sequence, ACF and PACF



Outline

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- **Separating between-person and within-person variance**
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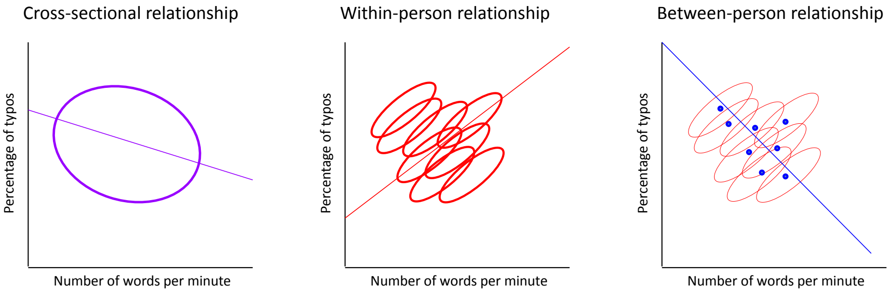
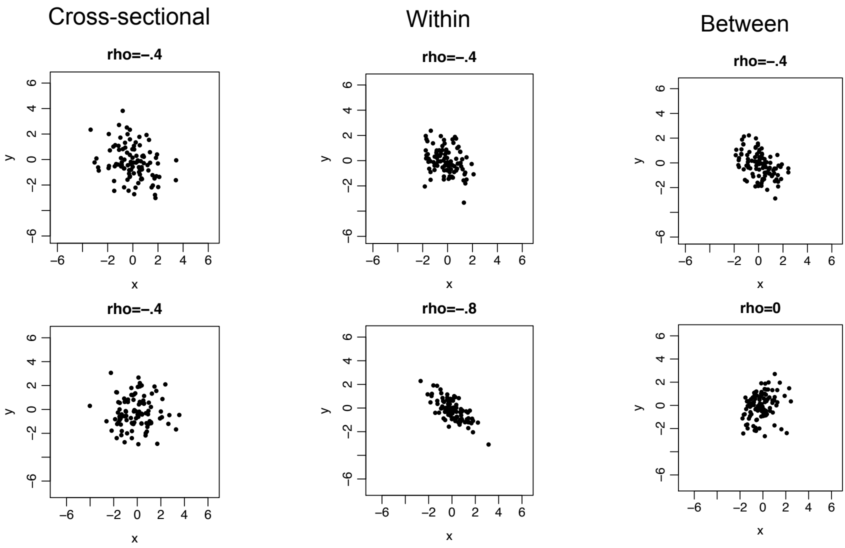
A fundamental problem in a nutshell



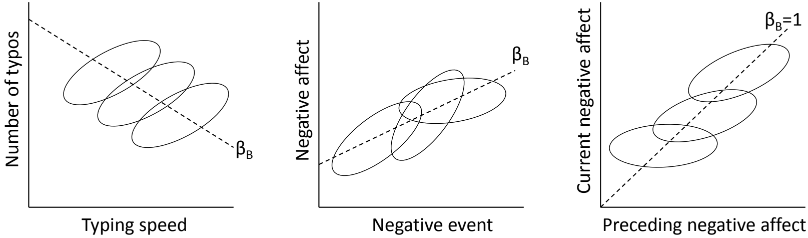
Taken from Hamaker (2012).

Three perspectives on data



Taken from Hamaker (2012).

Between-person differences in within-person slopes



Taken from Hamaker and Grasman (2014).

In conclusion: To study within-person processes we need

- (intensive) **longitudinal** data
- to **decompose** observed variance into within and between
- to consider **individual differences** in within-person dynamics

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Data: Daily measurements affect

Data come from the **COGITO study** of the MPI in Berlin; goal is to study aging using a younger and older sample.

Analyses here are based on Hamaker et al. (in preparation).

Characteristics of the **younger** and **older sample**:

- aged 20-31; aged 65-80
- 101 individuals; 103 individuals
- about 100 daily measurements of positive affect (PA) and negative affect (NA)

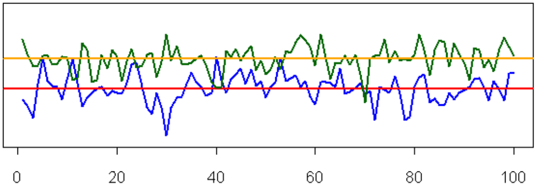
Decomposition

Decomposition into a between part and a within part

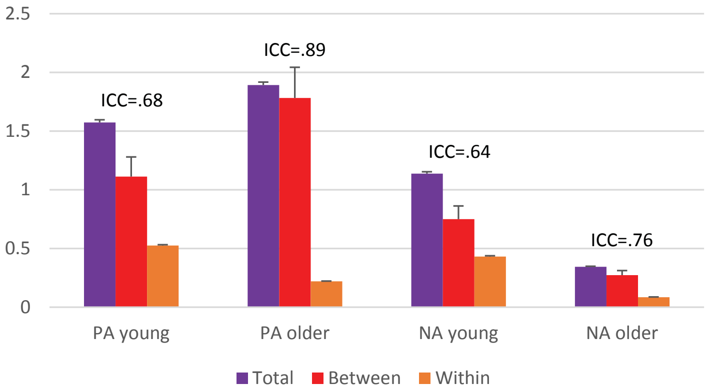
$$PA_{it} = \mu_{PA,i} + PA^*_{it}$$
$$NA_{it} = \mu_{NA,i} + NA^*_{it}$$

where

- $\mu_{PA,i}$ and $\mu_{NA,i}$ are the individual's **means** on PA and NA (i.e., baseline, trait, or equilibrium scores) $\Rightarrow$ between-person part
- PA^*_{it} and NA^*_{it} are the **within-person centered** (cluster-mean centered) scores $\Rightarrow$ within-person part



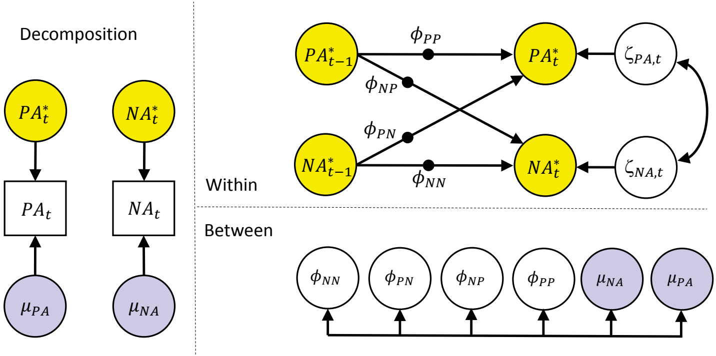
Total, between-, and within-person variance



Intraclass correlation:

$$\frac{\sigma^2_{between}}{\sigma^2_{between}+\sigma^2_{within}} = \frac{\sigma^2_{between}}{\sigma^2_{total}}$$

Bivariate model: Multilevel vector AR(1) model



Within-person level model

Lagged within-person model:

$$PA^*_{it} = \phi_{PP,i}PA^*_{i,t-1} + \phi_{PN,i}NA^*_{i,t-1} + \zeta_{PA,it}$$
$$NA^*_{it} = \phi_{NN,i}NA^*_{i,t-1} + \phi_{NP,i}PA^*_{i,t-1} + \zeta_{NA,it}$$

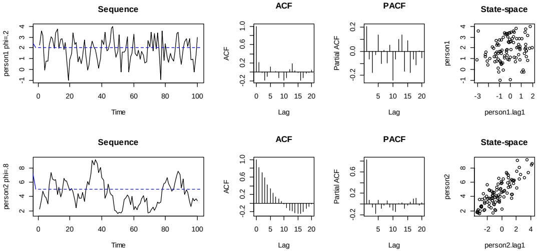
where

- $\phi_{PP,i}$ is the **autoregressive parameter** for PA (i.e., inertia, carry-over)
- $\phi_{NN,i}$ is the **autoregressive parameter** for NA (i.e., inertia, carry-over)
- $\phi_{PN,i}$ is the **cross-lagged parameter** for NA to PA (i.e., spill-over)
- $\phi_{NP,i}$ is the **cross-lagged parameter** for PA to NA (i.e., spill-over)
- $\zeta_{PA,it}$ is the **innovation** for PA (residual, disturbance, dynamic error)
- $\zeta_{NA,it}$ is the **innovation** for NA (residual, disturbance, dynamic error)

Parameters estimated at this level are the residual variances and covariance:

$$\begin{bmatrix}\zeta_{PA,it}\\ \zeta_{NA,it}\end{bmatrix} \sim MN\left[\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}\theta_{11} & \\ \theta_{21} & \theta_{22}\end{bmatrix}\right]$$

Autoregressive parameter (also known as inertia)



The AR parameter indicates how quickly a person recovers after being perturbed.

Affective inertia has been **empirically related to**

- neuroticism (+) and agreeableness (-) (Suls, Green & Hillis, 1998)
- concurrent depression (+) (Kuppens, Allen & Sheeber, 2010)
- future depression (+) (Kuppens, Sheeber, Yap, Whittle, Simmons & Allen, 2012)
- rumination (+) (Koval, Kuppens, Allen & Sheeber, 2012)
- self-esteem (-) (Houben, Van den Noortgate & Kuppens, 20150)
- life-satisfaction (-) (Houben et al., 2015)
- PA (-) and NA (+) (Houben et al., 2015)

Between-person level model

Between level: fixed and random effects

$$
\begin{bmatrix} \mu_{PA,i} \\ \mu_{NA,i} \\ \phi_{PP,i} \\ \phi_{PN,i} \\ \phi_{NP,i} \\ \phi_{NN,i} \end{bmatrix} = \begin{bmatrix} \gamma_{P} \\ \gamma_{N} \\ \gamma_{PP} \\ \gamma_{PN} \\ \gamma_{NP} \\ \gamma_{NN} \end{bmatrix} + \begin{bmatrix} u_{P,i} \\ u_{N,i} \\ u_{PP,i} \\ u_{PN,i} \\ u_{NP,i} \\ u_{NN,i} \end{bmatrix} \qquad \boldsymbol{u}_i \sim MN(\mathbf{0}, \boldsymbol{\Psi})
$$

Where:

- γ_P to γ_{NN} $\Rightarrow$ fixed effects
- $u_{P,i}$ to $u_{NN,i}$ $\Rightarrow$ random effects

Parameters estimated at this level are:

- 6 fixed effects (i.e., γ's)
- 6 variances for random effects (i.e., diagonal elements of $\boldsymbol{\Psi}$)
- 15 covariances between the random effects (i.e., off-diagonal elements in $\boldsymbol{\Psi}$)

Bivariate model: Mplus code

```
VARIABLE:
names       =   ID sessdate na1 na2 na3 na4 na5 na6 na7 na8 na9 na10
                pa1 pa2 pa3 pa4 pa5 pa6 pa7 pa8 pa9 pa10 sessionNr
                age_pre sex CESDpre CESDpost dayNA dayPA older;
cluster     =   ID;
usevar      =   dayPA dayNA;
lagged      =   dayPA(1) dayNA(1);
tinterval   =   sessdate(1);
missing     =   all(-999);

ANALYSIS:   TYPE IS TWOLEVEL random;
            estimator=bayes; proc = 2;
            fbiter= 5000; bseed = 2359;
            thin = 10;

MODEL:
    %WITHIN%
    p_pp | dayPA ON dayPA&1;
    p_pn | dayPA ON dayNA&1;
    p_np | dayNA ON dayPA&1;
    p_nn | dayNA ON dayNA&1;

    %BETWEEN%
    p_pp WITH p_pn-p_nn dayPA dayNA;
    p_pn WITH p_np-p_nn dayPA dayNA;
    p_np WITH p_nn dayPA dayNA;
    p_nn WITH dayPA dayNA;
    dayPA WITH dayNA;
```

Mplus results: Within-person (younger sample)

	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I. Lower 2.5%	Upper 2.5%	Significance
Within Level						
DAYNA WITH						
DAYPA	-0.069	0.004	0.000	-0.076	-0.061	*
Residual Variances						
DAYPA	0.414	0.006	0.000	0.403	0.426	*
DAYNA	0.302	0.004	0.000	0.294	0.311	*

Mplus results: Between-person (younger sample)

```
                                Posterior  One-Tailed         95% C.I.
                    Estimate       S.D.      P-Value   Lower 2.5%  Upper 2.5%  Significance

[...]

Between Level

[...]

 Means
    DAYPA              3.090       0.110      0.000       2.875       3.308      *
    DAYNA              0.977       0.077      0.000       0.826       1.128      *
    P_PP               0.334       0.026      0.000       0.283       0.387      *
    P_PN               0.050       0.022      0.016       0.006       0.093      *
    P_NP               0.038       0.015      0.006       0.008       0.068      *
    P_NN               0.370       0.027      0.000       0.315       0.423      *

 Variances
    DAYPA              1.178       0.189      0.000       0.886       1.618      *
    DAYNA              0.595       0.101      0.000       0.443       0.832      *
    P_PP               0.055       0.010      0.000       0.039       0.079      *
    P_PN               0.024       0.006      0.000       0.014       0.039      *
    P_NP               0.013       0.003      0.000       0.008       0.021      *
    P_NN               0.062       0.012      0.000       0.044       0.089      *
```

Comparing cross-lagged parameters

Standardization in multilevel models is a **tricky issue**.

Schuurman, Ferrer, Boer-Sonnenschein and Hamaker (2016) discuss four forms of **standardization in multilevel models**, using:

- total variance (i.e., grand standardization)
- between-person variance (i.e., between standardization)
- average within-person variance
- within-person variance (i.e., within standardization)

Conclusion: last form is most meaningful, as it **parallels standardizing when N=1**.

Standardized fixed effect should be the **average standardized within-person effect**.

Mplus standardized results (younger sample)

STDYX Standardization

Within-Level Standardized Estimates Averaged Over Clusters

	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I. Lower 2.5%	95% C.I. Upper 2.5%	Significance
P_PP | DAYPA ON						
DAYPA&1	0.335	0.011	0.000	0.312	0.358	*
P_PN | DAYPA ON						
DAYNA&1	0.034	0.013	0.006	0.008	0.059	*
P_NP | DAYNA ON						
DAYPA&1	0.038	0.011	0.000	0.017	0.059	*
P_NN | DAYNA ON						
DAYNA&1	0.370	0.012	0.000	0.347	0.394	*
DAYNA WITH						
DAYPA	-0.194	0.010	0.000	-0.213	-0.175	*
Residual Variances						
DAYPA	0.816	0.008	0.000	0.799	0.832	*
DAYNA	0.792	0.008	0.000	0.775	0.808	*

Mplus standardized results (younger sample)

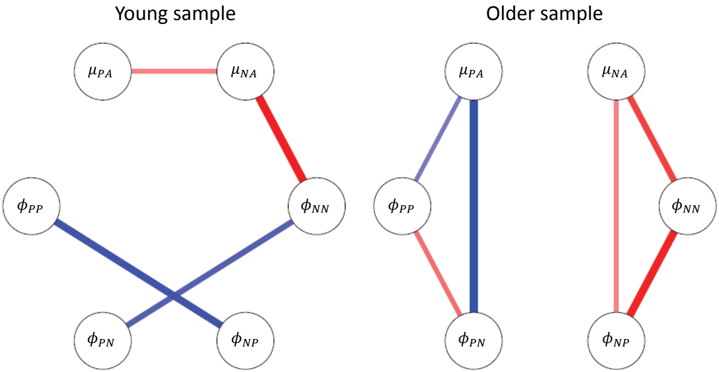
R-SQUARE

Within-Level R-Square Averaged Across Clusters

Variable	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I. Lower 2.5%	95% C.I. Upper 2.5%
DAYPA	0.184	0.008	0.000	0.168	0.201
DAYNA	0.208	0.008	0.000	0.192	0.225

Between-person level: Correlated random effects

To **represent the correlation matrices** of the 6 random effects in each group, we can use the network representation (with qgraph from Sacha Epskamp in R):



Including level 2 predictor and outcome

Depression was measured prior to the ILD phase and afterwards, using the CESD; we include these measures at the between-person level as a **predictor** and an **outcome**.

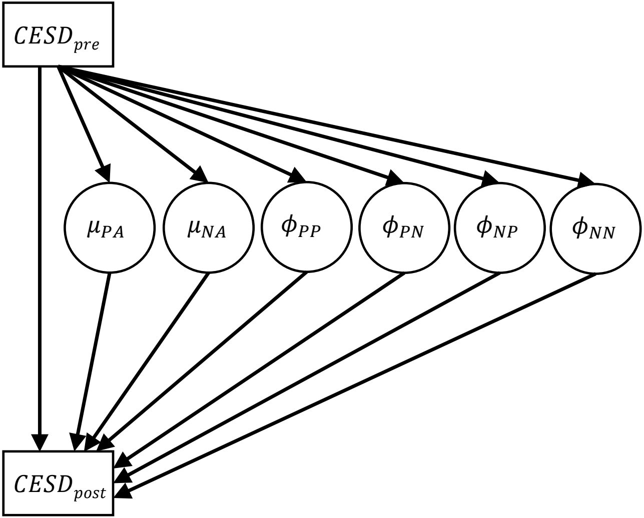
Between level: Including a level 2 predictor

$$
\begin{aligned}
\mu_{PA,i} &= \gamma_{00} + \gamma_{01}\, CESDpre_i + u_{0i} \\
\mu_{NA,i} &= \gamma_{10} + \gamma_{11}\, CESDpre_i + u_{1i} \\
\phi_{PP,i} &= \gamma_{20} + \gamma_{21}\, CESDpre_i + u_{2i} \\
\phi_{PN,i} &= \gamma_{30} + \gamma_{31}\, CESDpre_i + u_{3i} \\
\phi_{NN,i} &= \gamma_{40} + \gamma_{41}\, CESDpre_i + u_{4i} \\
\phi_{NP,i} &= \gamma_{50} + \gamma_{51}\, CESDpre_i + u_{5i}
\end{aligned}
$$

Between level: Including a level 2 outcome

$$
\begin{aligned}
CESDpost_i = \gamma_{60} &+ \gamma_{61}\, CESDpre_i + \gamma_{62}\mu_{PA,i} + \gamma_{63}\mu_{NA,i} \\
&+ \gamma_{64}\phi_{PP,i} + \gamma_{65}\phi_{PN,i} + \gamma_{66}\phi_{NN,i} + \gamma_{67}\phi_{NP,i} + u_{6i}
\end{aligned}
$$

Dynamic mediation model



Mplus input mediation model

```
VARIABLE:

names       = ID sessdate na1 na2 na3 na4 na5 na6 na7 na8 na9 na10
                pa1 pa2 pa3 pa4 pa5 pa6 pa7 pa8 pa9 pa10 sessionNr
                age_pre sex CESDpre CESDpost dayNA dayPA older;
cluster     =   ID;
usevar      =   dayPA dayNA CESDpre CESDpost;
between =       CESDpre CESDpost;
lagged      =   dayPA(1) dayNA(1);
tinterval   =   sessdate(1);
missing     =   all(-999);

DEFINE: CENTER CESDpre CESDpost (GRANDMEAN);
```

Mplus input mediation model

```
MODEL:
    %WITHIN%
    p_pp | dayPA ON dayPA&1;
    p_pn | dayPA ON dayNA&1;
    p_np | dayNA ON dayPA&1;
    p_nn | dayNA ON dayNA&1;

    %BETWEEN%
    p_pp-p_nn dayPA dayNA ON CESDpre (a1-a6);
    CESDpost ON p_pp-p_nn dayPA dayNA CESDpre (b1-b7);

  model constraint:
  new (ab_p_pp); ab_p_pp=a1*b1;
  new (ab_p_pn); ab_p_pn=a2*b2;
  new (ab_p_np); ab_p_np=a3*b3;
  new (ab_p_nn); ab_p_nn=a4*b4;
  new (ab_dayPA); ab_dayPA=a5*b5;
  new (ab_dayNA); ab_dayNA=a6*b6;
```

Note that the default here is that the residuals are **not correlated**.

Mplus output mediation model (younger sample)

	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I. Lower 2.5%	95% C.I. Upper 2.5%	Significance
[...]						
Between Level						
[...]						
Intercepts						
CESDPOST	0.104	0.136	0.223	-0.167	0.365	
DAYPA	3.088	0.103	0.000	2.888	3.293	*
DAYNA	0.989	0.076	0.000	0.844	1.146	*
P_PP	0.338	0.024	0.000	0.289	0.386	*
P_PN	0.031	0.020	0.057	-0.008	0.071	
P_NP	0.035	0.014	0.006	0.007	0.062	*
P_NN	0.376	0.024	0.000	0.329	0.423	*
Residual Variances						
CESDPOST	0.067	0.012	0.000	0.048	0.095	*
DAYPA	1.049	0.158	0.000	0.798	1.416	*
DAYNA	0.517	0.091	0.000	0.377	0.729	*
P_PP	0.045	0.008	0.000	0.032	0.064	*
P_PN	0.019	0.005	0.000	0.011	0.030	*
P_NP	0.010	0.003	0.000	0.005	0.016	*
P_NN	0.043	0.008	0.000	0.031	0.062	*
New/Additional Parameters						
AB_P_PP	0.010	0.025	0.266	-0.028	0.076	
AB_P_PN	-0.002	0.032	0.439	-0.074	0.062	
AB_P_NP	-0.004	0.037	0.401	-0.089	0.067	
AB_P_NN	0.195	0.070	0.000	0.081	0.359	*
AB_DAYPA	0.049	0.035	0.029	-0.001	0.135	
AB_DAYNA	0.028	0.043	0.234	-0.052	0.119	

Mplus output mediation model (older sample)

```
                                Posterior  One-Tailed         95% C.I.
                    Estimate       S.D.      P-Value   Lower 2.5%  Upper 2.5%  Significance
[...]
Between Level
[...]

 Intercepts
    CESDPOST           0.015       0.113      0.448      -0.210       0.236
    DAYPA              4.566       0.120      0.000       4.336       4.796      *
    DAYNA              0.313       0.052      0.000       0.210       0.417      *
    P_PP               0.421       0.026      0.000       0.370       0.472      *
    P_PN               0.133       0.039      0.000       0.057       0.212      *
    P_NP               0.016       0.017      0.167      -0.018       0.051
    P_NN               0.239       0.027      0.000       0.185       0.291      *

 Residual Variances
    CESDPOST           0.039       0.006      0.000       0.029       0.053      *
    DAYPA              1.416       0.221      0.000       1.079       1.918      *
    DAYNA              0.269       0.041      0.000       0.203       0.365      *
    P_PP               0.056       0.010      0.000       0.039       0.079      *
    P_PN               0.083       0.021      0.000       0.051       0.131      *
    P_NP               0.024       0.004      0.000       0.018       0.035      *
    P_NN               0.051       0.009      0.000       0.037       0.072      *

New/Additional Parameters
    AB_P_PP            0.005       0.016      0.302      -0.018       0.049
    AB_P_PN           -0.004       0.025      0.396      -0.061       0.045
    AB_P_NP            0.012       0.027      0.268      -0.035       0.076
    AB_P_NN           -0.036       0.038      0.112      -0.130       0.025
    AB_DAYPA           0.028       0.038      0.209      -0.042       0.110
    AB_DAYNA           0.027       0.036      0.194      -0.040       0.108
```

Random variance (cf. Jongerling et al., 2015)

Within level: AR(1) with random ϕ_i

$$NA^*_{it} = \phi_i NA^*_{i,t-1} + \zeta_{it} \qquad\qquad \zeta_{it} \sim N(0, \sigma^2)$$

Where ζ is the **innovation**, consisting of:

- external influences
- reactivity to external influences

Reasons to assume **individual differences** for σ^2:

- individuals may differ with respect to the **variability in exposure** to external factors
- individuals may differ with respect to their **reactivity** to external influences (see reward experience and stress sensitivity research)

Hence, we allow for a **random innovation variance** using a log normal distribution.

Random innovation variance: Univariate model

Within level: AR(1) with random ϕ_i

$$NA^*_{it} = \phi_i NA^*_{i,t-1} + \zeta_{it} \qquad \zeta_{it} \sim N(0, \sigma^2_i)$$

Between level: fixed and random effects

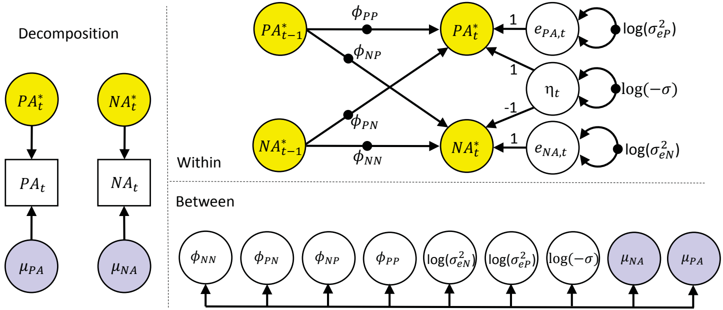
$$\begin{aligned} \mu_i &= \gamma_\mu + u_{0i} \\ \phi_i &= \gamma_\phi + u_{1i} \\ \log(\sigma^2_i) &= \gamma_{\log(\sigma^2)} + u_{2i} \end{aligned} \qquad \begin{bmatrix} u_{0i} \\ u_{1i} \\ u_{2i} \end{bmatrix} \sim MN\left[\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{11} & & \\ \psi_{21} & \psi_{22} & \\ \psi_{31} & \psi_{32} & \psi_{33} \end{bmatrix}\right]$$

```
MODEL:
    %WITHIN%
    p_nn | dayNA ON dayNA&1;
    logRVNA | dayNA;

    %BETWEEN%
    p_nn WITH logRVNA dayNA;
    logRVNA WITH dayNA;
```

Bivariate model: Random innovation variance

In the bivariate case, we want **random innovation variances** AND **random innovation covariance**.

The latter is modeled with an additional factor η_t:



Where:

- -η_t is the shared part (we assume a negative covariance)
- $e_{PA,t}$ and $e_{NA,t}$ are the unique parts

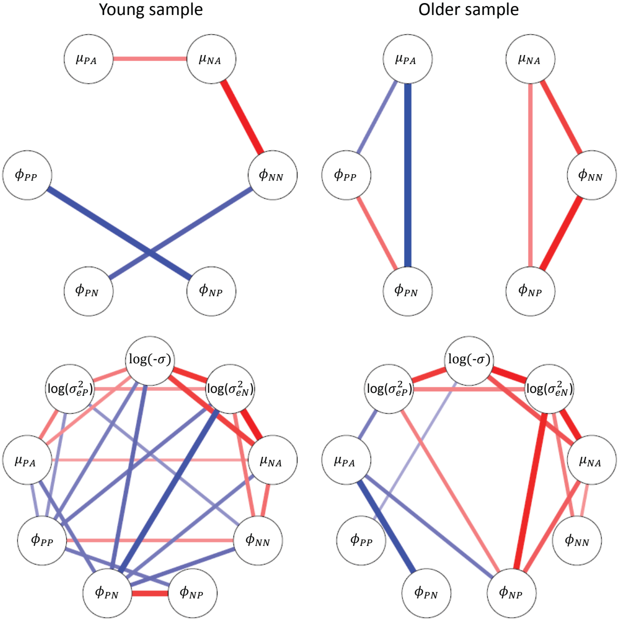
Mplus code

```
MODEL:
   %WITHIN%
   p_pp | dayPA ON dayPA&1;
   p_pn | dayPA ON dayNA&1;
   p_np | dayNA ON dayPA&1;
   p_nn | dayNA ON dayNA&1;

   logvarPA | dayPA; ! RANDOM UNIQUE INNOVATION VARIANCE
   logvarNA | dayNA; ! RANDOM UNIQUE INNOVATION VARIANCE

   Cov BY dayPA@1 dayNA@-1; ! COMMON INNOVATION VARIANCE
   logCov | Cov; ! RANDOM COMMON INNOVATION VARIANCE

   %BETWEEN%
   p_pp WITH p_pn-p_nn logvarPA logvarNA logCov dayPA dayNA;
   p_pn WITH p_np-p_nn logvarPA logvarNA logCov dayPA dayNA;
   p_np WITH p_nn logvarPA logvarNA logCov dayPA dayNA;
   p_nn WITH logvarPA logvarNA logCov dayPA dayNA;
   logvarPA WITH logvarNA logCov dayPA dayNA;
   logvarNA WITH logCov dayPA dayNA;
   logCov WITH dayPA dayNA;
   dayPA WITH dayNA;
```

Mplus results (younger sample)

	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I. Lower 2.5%	95% C.I. Upper 2.5%	Significance
Within Level						
COV BY						
DAYPA	1.000	0.000	0.000	1.000	1.000	
DAYNA	-1.000	0.000	0.000	-1.000	-1.000	
Between Level						
[...]						
Means						
DAYPA	3.095	0.115	0.000	2.865	3.315	*
DAYNA	0.972	0.080	0.000	0.817	1.128	*
P_PP	0.373	0.026	0.000	0.322	0.423	*
P_PN	0.080	0.023	0.000	0.036	0.126	*
P_NP	0.030	0.012	0.006	0.007	0.054	*
P_NN	0.396	0.028	0.000	0.342	0.450	*
LOGVARPA	-1.330	0.087	0.000	-1.509	-1.160	*
LOGVARNA	-2.038	0.143	0.000	-2.326	-1.767	*
LOGCOV	-3.275	0.159	0.000	-3.599	-2.973	*
Variances						
DAYPA	1.265	0.209	0.000	0.951	1.754	*
DAYNA	0.605	0.103	0.000	0.447	0.847	*
P_PP	0.057	0.011	0.000	0.040	0.083	*
P_PN	0.029	0.008	0.000	0.018	0.047	*
P_NP	0.007	0.002	0.000	0.004	0.012	*
P_NN	0.065	0.012	0.000	0.047	0.094	*
LOGVARPA	0.692	0.124	0.000	0.501	0.994	*
LOGVARNA	1.900	0.328	0.000	1.395	2.650	*
LOGCOV	1.912	0.377	0.000	1.330	2.800	*

Correlated random effects (before and now)



Mediation model with random innovation variances and covariance

```
MODEL:
   %WITHIN%
   p_pp | dayPA ON dayPA&1;
   p_pn | dayPA ON dayNA&1;
   p_np | dayNA ON dayPA&1;
   p_nn | dayNA ON dayNA&1;

   logvarPA | dayPA; ! RANDOM UNIQUE INNOVATION VARIANCE
   logvarNA | dayNA; ! RANDOM UNIQUE INNOVATION VARIANCE

   Cov BY dayPA@1 dayNA@-1; ! COMMON INNOVATION VARIANCE
   logCov | Cov; ! RANDOM COMMON INNOVATION VARIANCE

   %BETWEEN%
   p_pp-p_nn dayPA dayNA ON CESDpre (a1-a6);
   logvarPA logvarNA logCov ON CESDpre (a7-a9);
   CESDpost ON p_pp-p_nn dayPA dayNA logvarPA logvarNA logCov CESDpre (b1-b10);

model constraint:
 new (ab_p_pp); ab_p_pp=a1*b1;
 new (ab_p_pn); ab_p_pn=a2*b2;
 new (ab_p_np); ab_p_np=a3*b3;
 new (ab_p_nn); ab_p_nn=a4*b4;
 new (ab_dayPA); ab_dayPA=a5*b5;
 new (ab_dayNA); ab_dayNA=a6*b6;
 new (ab_lvPA); ab_lvPA=a7*b7;
 new (ab_lvNA); ab_lvNA=a8*b8;
 new (ab_lCov); ab_lCov=a9*b9;
```

Mplus results

Effect	Younger	Older
direct	**0.290 [0.062,0.522]**	**0.585 [0.076,1.206]**
mediated by μ_{PA}	*0.058 [-0.011,0.154]*	*0.054 [-0.018,0.147]*
mediated by μ_{NA}	*0.024 [-0.062,0.130]*	*0.011 [-0.022,0.070]*
mediated by ϕ_{PP}	*0.003 [-0.032,0.050]*	*0.003 [-0.020,0.043]*
mediated by ϕ_{PN}	*0.000 [-0.053,0.061]*	*-0.003 [-0.106,0.097]*
mediated by ϕ_{NP}	*-0.019 [-0.178,0.087]*	*-0.048 [-0.691,0.470]*
mediated by ϕ_{NN}	**0.127 [0.036,0.258]**	*-0.011 [-0.069,0.020]*
mediated by $log(\sigma^2_{eP})$	*0.000 [-0.059,0.055]*	*-0.046 [-0.127,0.007]*
mediated by $log(\sigma^2_{eN})$	*-0.009 [-0.103,0.076]*	*0.079 [-0.015,0.212]*
mediated by $log(-\sigma)$	**0.072 [0.004,0.185]**	*0.029 [-0.035,0.122]*

Hence:

- higher CESDpre is associated with higher CESDpost (both samples)
- higher CESDpre predicts more carry-over in NA, which subsequently predicts higher CESDpost (younger sample)
- higher CESDpre predicts higher $log(-\sigma)$, which subsequently predicts higher CESDpost (younger sample)

Mediation through the random common variance

For the younger sample we have:

```
                                 Posterior  One-Tailed         95% C.I.
                      Estimate      S.D.      P-Value   Lower 2.5%  Upper 2.5%  Significance
[...]
Between Level
[...]

 LOGCOV     ON
    CESDPRE              0.986       0.426      0.011       0.142       1.796       *

 CESDPOST   ON
 [...]
    LOGCOV               0.080       0.031      0.006       0.017       0.143       *
```

Considering three levels of CESDpre (SD of CESDpre is 0.35):

- +2SD CESDpre: $log(-\sigma) = -2.69 \rightarrow -\sigma = 0.07 \rightarrow \sigma = -0.07$
- ±0SD CESDpre: $log(-\sigma) = -3.39 \rightarrow -\sigma = 0.03 \rightarrow \sigma = -0.03$
- -2SD CESDpre: $log(-\sigma) = 4.09 \rightarrow -\sigma = 0.02 \rightarrow \sigma = -0.02$

Conclusion: Higher CESDpre is associated with more negative common variance (i.e., covariance).

Results younger sample

```
                              Posterior  One-Tailed        95% C.I.
                   Estimate      S.D.      P-Value   Lower 2.5%  Upper 2.5%  Significance
[...]

Between Level

 P_PP       ON
    CESDPRE      -0.030       0.069      0.328      -0.162       0.109

 P_PN       ON
    CESDPRE      -0.006       0.055      0.452      -0.116       0.101

 P_NP       ON
    CESDPRE       0.054       0.034      0.057      -0.014       0.119

 P_NN       ON
    CESDPRE       0.241       0.069      0.001       0.102       0.374      *

 LOGVARPA   ON
    CESDPRE      -0.535       0.239      0.014      -1.000      -0.055      *

 LOGVARNA   ON
    CESDPRE       1.301       0.372      0.000       0.576       2.019      *

 LOGCOV     ON
    CESDPRE       0.986       0.426      0.011       0.142       1.796      *

 CESDPOST   ON
    P_PP         -0.210       0.187      0.128      -0.576       0.162
    P_PN         -0.344       0.331      0.146      -0.995       0.306
    P_NP         -0.548       0.973      0.275      -2.575       1.279
    P_NN          0.553       0.174      0.001       0.220       0.898      *
    LOGVARPA      0.002       0.047      0.487      -0.086       0.095
    LOGVARNA     -0.008       0.033      0.399      -0.074       0.056
    LOGCOV        0.080       0.031      0.006       0.017       0.143      *
```

Results younger sample (continued)

```
                          Posterior  One-Tailed      95% C.I.
              Estimate      S.D.      P-Value   Lower 2.5%  Upper 2.5%  Significance
[...]
Between Level
[...]

 DAYPA      ON
    CESDPRE     -0.509      0.308      0.050      -1.117       0.097

 DAYNA      ON
    CESDPRE      0.782      0.231      0.000       0.343       1.246      *

 CESDPOST   ON
    DAYPA       -0.121      0.033      0.000      -0.187      -0.057      *
    DAYNA        0.034      0.057      0.274      -0.077       0.145
    CESDPRE      0.290      0.115      0.005       0.062       0.522      *
```

Results older sample

	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I. Lower 2.5%	95% C.I. Upper 2.5%	Significance
[...]						
Between Level						
P_PP ON						
CESDPRE	0.063	0.101	0.261	-0.134	0.264	
P_PN ON						
CESDPRE	0.203	0.114	0.037	-0.023	0.429	
P_NP ON						
CESDPRE	0.048	0.019	0.008	0.010	0.088	*
P_NN ON						
CESDPRE	0.090	0.102	0.184	-0.114	0.291	
LOGVARPA ON						
CESDPRE	1.117	0.393	0.003	0.361	1.897	*
LOGVARNA ON						
CESDPRE	2.356	0.655	0.000	1.102	3.677	*
LOGCOV ON						
CESDPRE	1.635	0.608	0.002	0.424	2.814	*
CESDPOST ON						
P_PP	0.093	0.098	0.173	-0.099	0.286	
P_PN	-0.030	0.209	0.441	-0.433	0.408	
P_NP	-1.169	28.837	0.317	-12.494	8.813	
P_NN	-0.168	0.102	0.053	-0.368	0.029	
LOGVARPA	-0.045	0.025	0.042	-0.095	0.006	
LOGVARNA	0.035	0.021	0.045	-0.007	0.076	
LOGCOV	0.020	0.021	0.158	-0.021	0.060	

Results older sample (continued)

```
                                  Posterior  One-Tailed        95% C.I.
                      Estimate       S.D.      P-Value    Lower 2.5%  Upper 2.5%  Significance
[...]
Between Level
[...]

 DAYPA      ON
    CESDPRE            -2.003       0.490      0.000      -2.940      -1.021       *

 DAYNA      ON
    CESDPRE             0.181       0.205      0.192      -0.234       0.578

 CESDPOST   ON
    DAYPA              -0.028       0.019      0.070      -0.065       0.009
    DAYNA               0.087       0.053      0.047      -0.015       0.192
    CESDPRE             0.585       1.292      0.021       0.076       1.206       *
```

Advantages of using DSEM in Mplus (thus far)

Compared to standard multilevel software:

- multiple outcome variables (with correlated residuals)
- outcomes at between-person level
- person-mean centering integral part of model estimation

Hamaker and Grasman — Centering in a multilevel autoregressive model

Table 4 | Bias and coverage rates for fixed autoregressive parameter ϕ in multilevel autoregressive model under diverse scenarios.

AR parameter	Sample size		Bias				$CR_{0.95}$			
	N	T	NC	$C(\bar{y}_{.i})$	$C(\hat{\mu}_i)$	$C(\mu_i)$	NC	$C(\bar{y}_{.i})$	$C(\hat{\mu}_i)$	$C(\mu_i)$
$\phi_i \sim N(0.3, 0.1)$	20	20	0.002	−0.072	−0.069	−0.068	0.928	0.762	0.785	0.787
		50	0.000	−0.027	−0.027	−0.026	0.940	0.900	0.901	0.898
		100	0.000	−0.013	−0.013	−0.013	0.932	0.932	0.932	0.932
	50	20	0.005	−0.071	−0.069	−0.067	0.893	0.480	0.512	0.518
		50	0.001	−0.027	−0.026	−0.026	0.936	0.800	0.804	0.805
		100	0.000	−0.013	−0.013	−0.013	0.946	0.902	0.902	0.903
	100	20	0.006	−0.070	−0.068	−0.066	0.892	0.196	0.227	0.242
		50	0.001	−0.027	−0.027	−0.027	0.930	0.623	0.630	0.637
		100	0.000	−0.013	−0.013	−0.013	0.930	0.851	0.854	0.851

Advantages of using DSEM in Mplus

All the models ran here, could also be estimated using other **Bayesian software** (e.g., WinBUGS, jags, and stan).

In comparison, the **advantages of Mplus** are:

- **easy to use** due to tailor-made code
- **default uninformative priors** for parameters (even for small variances)
- **fast** (which makes a difference in case of Bayes)

Other recent developments:

- **ctsem** in R: Allows for continuous time modeling
- **open Mx** in R

Outline

- Modeling the dynamics of ILD
- Separating between-person and within-person variance
- Application 1: Daily negative affect and depressive symptomatology
- **Application 2: Intervention study with ESM**
- Application 3: Dyadic daily diary data
- Application 4: Latent AR(1) model
- Discussion

Intervention study with ESM

When **ESM** is used in a **randomized controlled trial**, we can investigate whether treatment affects:

- means
- dynamics (e.g., autoregression)
- variability

Here we use data from individuals with a **history of depression** and current residual depressive symptoms (Geschwind et al., 2011).

Each ESM period consisted of 6 days, 10 beeps per day.

We analyze data from 117 participants; 56 received a **mindfulness training** between the two phases, and 61 served as **controls**.

Treatment effect on the within-person mean

We use NA_{1it} and NA_{2it} as **two separate variables**!

Decomposition into a between part and a within part

$$\begin{array}{ll} \text{Pre-treatment phase:} & NA_{1it} = \mu_{1i} + NA^*_{1it} \\ \text{Post-treatment phase:} & NA_{2it} = \mu_{2i} + NA^*_{2it} \end{array}$$

Between level

$$\mu_{1i} = \gamma_{00} + \gamma_{01} Group_i + u_{1i}$$
$$\mu_{2i} = \gamma_{10} + \mu_{1i} + \gamma_{11} Group_i + u_{2i}$$

- γ_{01} is the **initial difference** between the groups
- γ_{10} is the **effect of time**
- γ_{11} is the **effect of treatment**

Note: $\mu_{2i} - \mu_{1i} = \gamma_{10} + \gamma_{11} Group_i + u_{2i}$.

Mplus input

```
MODEL:
    %WITHIN%
    na_pre WITH na_post@0;

    %BETWEEN%
    na_pre ON Group;
    na_post ON na_pre@1 Group;
    na_pre WITH na_post;
```

Note: When NA_{1it} is observed, NA_{2it} is missing, and vice versa; hence, we fix their within-person **covariance to zero**.

Mplus results

	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I. Lower 2.5%	95% C.I. Upper 2.5%	Significance
Within Level						
NA_PRE WITH						
NA_POST	0.000	0.000	1.000	0.000	0.000	
Variances						
NA_PRE	0.639	0.012	0.000	0.616	0.662	*
NA_POST	0.483	0.009	0.000	0.466	0.501	*
Between Level						
NA_PRE ON						
GROUP	-0.005	0.136	0.484	-0.292	0.249	
NA_POST ON						
NA_PRE	1.000	0.000	0.000	1.000	1.000	
GROUP	-0.320	0.108	0.002	-0.539	-0.112	*
NA_PRE WITH						
NA_POST	-0.157	0.046	0.000	-0.262	-0.082	*
Intercepts						
NA_PRE	2.019	0.095	0.000	1.837	2.210	*
NA_POST	0.006	0.077	0.472	-0.148	0.155	
Residual Variances						
NA_PRE	0.524	0.078	0.000	0.402	0.706	*
NA_POST	0.324	0.050	0.000	0.247	0.439	*

Treatment effect on autoregression

Within level: AR(1) processes

$$\begin{aligned} \text{Pre-treatment phase:} \quad & NA^*_{1it} = \phi_{1i} NA^*_{1it} + \zeta_{1it} \\ \text{Post-treatment phase:} \quad & NA^*_{2it} = \phi_{2i} NA^*_{2it} + \zeta_{2it} \end{aligned}$$

Between level: Pre-treatment phase

$$\mu_{1i} = \gamma_{00} + \gamma_{01} Group_i + u_{0i}$$
$$\phi_{1i} = \gamma_{10} + \gamma_{11} Group_i + u_{1i}$$

We expect γ_{01} and γ_{11} to be zero.

Between level: Post-treatment phase

$$\mu_{2i} = \gamma_{20} + \mu_{1i} + \gamma_{21} Group_i + u_{2i}$$
$$\phi_{2i} = \gamma_{30} + \phi_{1i} + \gamma_{31} Group_i + u_{3i}$$

Where: γ_{20} and γ_{30} represent the **effects of time** and: γ_{21} and γ_{31} represent the **effects of treatment**

Mplus results (all effects random)

```
 Between Level

  PHI2       ON
     PHI1             1.000       0.000       0.000       1.000       1.000

  PHI1       ON
     GROUP            0.052       0.047       0.130      -0.039       0.142

  PHI2       ON
     GROUP           -0.077       0.066       0.119      -0.209       0.057

  NA_PRE     ON
     GROUP           -0.079       0.134       0.284      -0.340       0.183

  NA_POST    ON
     NA_PRE           1.000       0.000       0.000       1.000       1.000
     GROUP           -0.246       0.105       0.010      -0.457      -0.038      *

  Intercepts
     NA_PRE           2.008       0.092       0.000       1.831       2.190      *
     NA_POST         -0.005       0.071       0.470      -0.148       0.136
     PHI1             0.454       0.034       0.000       0.390       0.522      *
     PHI2            -0.092       0.047       0.022      -0.185      -0.004      *

  Residual Variances
     NA_PRE           0.450       0.069       0.000       0.337       0.598      *
     NA_POST          0.247       0.044       0.000       0.171       0.342      *
     PHI1             0.040       0.008       0.000       0.027       0.059      *
     PHI2             0.082       0.018       0.000       0.053       0.121      *
```

Mplus results (with fixed change in ϕ)

Between Level

PHI2 ON						
PHI1	1.000	0.000	0.000	1.000	1.000	
PHI1 ON						
GROUP	0.075	0.049	0.053	-0.014	0.174	
PHI2 ON						
GROUP	-0.070	0.033	0.014	-0.137	-0.005	*
NA_PRE ON						
GROUP	-0.071	0.132	0.302	-0.327	0.192	
NA_POST ON						
NA_PRE	1.000	0.000	0.000	1.000	1.000	
GROUP	-0.247	0.105	0.010	-0.454	-0.043	*
Intercepts						
NA_PRE	2.012	0.090	0.000	1.837	2.194	*
NA_POST	-0.010	0.071	0.442	-0.152	0.133	
PHI1	0.425	0.034	0.000	0.356	0.491	*
PHI2	-0.019	0.022	0.199	-0.062	0.026	
Residual Variances						
NA_PRE	0.458	0.069	0.000	0.344	0.615	*
NA_POST	0.261	0.044	0.000	0.188	0.360	*
PHI1	0.050	0.009	0.000	0.035	0.070	*
PHI2	0.001	0.000	0.000	0.001	0.001	

Including a level 1 predictor

Let $UnPl_{1it}$ and $UnPl_{2it}$ be variables for phases 1 and 2, that indicate whether something emotionally charged happened since the previous beep (positive scores is Pleasant event, negative score is Unpleasant event).

Within level

$$\begin{aligned} \text{Pre-treatment phase:} \quad & NA^*_{1it} = \phi_{1i} NA^*_{1it} + \beta_{1i} UnPl^*_{1it} + \zeta_{1it} \\ \text{Post-treatment phase:} \quad & NA^*_{2it} = \phi_{2i} NA^*_{2it} + \beta_{2i} UnPl^*_{2it} + \zeta_{2it} \end{aligned}$$

where:

- ϕ_{1i} and ϕ_{2i} represent carry-over
- β_{1i} and β_{2i} represent reactivity/sensitivity

Including a level 1 predictor

At between level we include Group as predictor for pre-treatment phase:

Between level: Pre-treatment phase

$$\mu_{1i} = \gamma_{00} + \gamma_{01} Group_i + u_{0i}$$
$$\phi_{1i} = \gamma_{10} + \gamma_{11} Group_i + u_{1i}$$
$$\beta_{1i} = \gamma_{20} + \gamma_{21} Group_i + u_{2i}$$

where γ_{00}, γ_{10}, and γ_{20} are expected to be zero.

For the post-treatment phase, we model the change in mean, carry-over, and reactivity:

Between level: Post-treatment phase

$$\mu_{2i} = \gamma_{40} + \mu_{1i} + \gamma_{41} Group_i + u_{4i}$$
$$\phi_{2i} = \gamma_{50} + \phi_{1i} + \gamma_{51} Group_i + u_{5i}$$
$$\beta_{2i} = \gamma_{60} + \beta_{1i} + \gamma_{61} Group_i + u_{6i}$$

where

- γ_{40}, γ_{50}, and γ_{60} represent **change due to time**
- γ_{41}, γ_{51}, and γ_{61} represent **treatment effect**

Mplus input: Centering within predictors

```
VARIABLE:

names       =   ID Time PrePost Group
                pa_pre pa_post na_pre na_post
                PDLA_pre PDLA_post UnPl_pre UnPl_post
                ham_pre ham_post ;
cluster     =   ID;
usevar      =   na_pre na_post UnPl_pre UnPl_post Group;
lagged      =   na_pre(1) na_post(1);
within      =   UnPl_pre UnPl_post;
between     =   Group;
tinterval   =   Time(1);
missing     =   all(-999);

DEFINE:  center UnPl_pre UnPl_post (groupmean);

ANALYSIS:   TYPE IS TWOLEVEL random; estimator=bayes;
            proc = 2; biter= (2000); bseed = 5229;
            thin = 10;
```

Mplus input: Within and between model

Note: The within-person predictor has missings; by asking for the variances, Mplus treats it as a y-variable, which is allowed to have missings.

```
MODEL:
    %WITHIN%
    phi1 | na_pre ON na_pre&1;
    beta1 | na_pre ON UnPl_pre;
    phi2 | na_post ON na_post&1;
    beta2 | na_post ON UnPl_post;

    na_pre-UnPl_post WITH na_post-UnPl_post@0;
    UnPl_pre; UnPl_post;

    %BETWEEN%
    na_pre phi1 beta1 ON Group;
    na_post ON na_pre@1 Group;
    phi2 ON phi1@1 Group;
    beta2 ON beta1@1 Group;
```

Mplus output: Regressions at Between level

```
Between Level

 PHI2       ON
    PHI1              1.000       0.000       0.000       1.000       1.000

 BETA2      ON
    BETA1             1.000       0.000       0.000       1.000       1.000

 PHI1       ON
    GROUP             0.050       0.046       0.119      -0.035       0.144

 BETA1      ON
    GROUP             0.001       0.019       0.470      -0.034       0.041

 PHI2       ON
    GROUP            -0.077       0.068       0.123      -0.214       0.053

 BETA2      ON
    GROUP            -0.016       0.026       0.264      -0.069       0.032

 NA_PRE     ON
    GROUP            -0.070       0.134       0.297      -0.340       0.180

 NA_POST    ON
    NA_PRE            1.000       0.000       0.000       1.000       1.000
    GROUP            -0.255       0.105       0.007      -0.463      -0.059      *
```

Group only has an effect on the change in the mean (i.e., $\mu_{2i} - \mu_{1i}$).

Mplus output: Intercepts and random effects

Intercepts						
NA_PRE	2.012	0.091	0.000	1.835	2.189	*
NA_POST	-0.014	0.071	0.422	-0.155	0.126	
PHI1	0.423	0.033	0.000	0.357	0.487	*
BETA1	-0.123	0.013	0.000	-0.150	-0.097	*
PHI2	-0.082	0.047	0.039	-0.173	0.011	
BETA2	0.005	0.018	0.388	-0.027	0.041	
Residual Variances						
NA_PRE	0.466	0.070	0.000	0.355	0.632	*
NA_POST	0.268	0.042	0.000	0.199	0.359	*
PHI1	0.038	0.008	0.000	0.026	0.056	*
BETA1	0.006	0.001	0.000	0.004	0.009	*
PHI2	0.078	0.016	0.000	0.051	0.114	*
BETA2	0.008	0.003	0.000	0.005	0.015	*

Conclusion:

- means of μ_{1i}, ϕ_{1i}, and β_{1i} deviate from zero
- no change due to time (intercepts for μ_{2i}, ϕ_{2i}, and β_{2i} are zero)

Including a level 2 predictor

Let Ham_{1i} and Ham_{2i} be depression scores for phases 1 and 2; these were obtained with the Hamilton depression scale prior to each ESM episode.

Within level

Pre-treatment phase: $NA^*_{1it} = \phi_{1i} NA^*_{1it} + \beta_{1i} UnPl^*_{1it} + \zeta_{1it}$

Post-treatment phase: $NA^*_{2it} = \phi_{2i} NA^*_{2it} + \beta_{2i} UnPl^*_{2it} + \zeta_{2it}$

where:

- ϕ_{1i} and ϕ_{2i} represent carry-over
- β_{1i} and β_{2i} represent reactivity/sensitivity

Including a level 2 predictor (pre-treatment)

At between level we include Group as predictor for pre-treatment phase:

Between level: Pre-treatment phase

$$\mu_{1i} = \gamma_{00} + \gamma_{01} Group_i + \gamma_{02} Ham_{1i} + u_{0i}$$
$$\phi_{1i} = \gamma_{10} + \gamma_{11} Group_i + \gamma_{12} Ham_{1i} + u_{1i}$$
$$\beta_{1i} = \gamma_{20} + \gamma_{21} Group_i + \gamma_{22} Ham_{1i} + u_{2i}$$
$$Ham_{1i} = \gamma_{30} + \gamma_{31} Group_i + u_{3i}$$

where

- γ_{01}, γ_{11}, γ_{21}, and γ_{31} are expected to be zero
- γ_{02} is expected to be positive
- γ_{12} is expected to be positive
- γ_{22} is expected to be non-zero

Including a level 2 predictor (post-treatment)

For the post-treatment phase, we model the change in mean, carry-over, reactivity, and depression score:

Between level: Post-treatment phase

$$\mu_{2i} = \gamma_{50} + \mu_{1i} + \gamma_{51} Group_i + \gamma_{52} Ham_{2i} + u_{5i}$$
$$\phi_{2i} = \gamma_{60} + \phi_{1i} + \gamma_{61} Group_i + \gamma_{62} Ham_{2i} + u_{6i}$$
$$\beta_{2i} = \gamma_{70} + \beta_{1i} + \gamma_{71} Group_i + \gamma_{72} Ham_{2i} + u_{7i}$$
$$Ham_{2i} = \gamma_{80} + Ham_{i1} + \gamma_{81} Group_i + u_{8i}$$

where

- γ_{50}, γ_{60}, γ_{70}, and γ_{80} represent **change due to time**
- γ_{51}, γ_{61}, γ_{71}, and γ_{81} represent **direct treatment effect**
- γ_{52}, γ_{62}, and γ_{72} represent change predicted by depression score
- $\gamma_{81} * \gamma_{52}$ treatment effect on change in mean mediated through depression
- $\gamma_{81} * \gamma_{62}$ treatment effect on change in carry-over mediated through depression
- $\gamma_{81} * \gamma_{72}$ treatment effect on change in reactivity mediated through depression

Mediation of Group

Between level: Post-treatment phase

$$\mu_{2i} = \gamma_{50} + \mu_{1i} + \gamma_{51} Group_i + \gamma_{52} Ham_{2i} + u_{5i}$$
$$\phi_{2i} = \gamma_{60} + \phi_{1i} + \gamma_{61} Group_i + \gamma_{62} Ham_{2i} + u_{6i}$$
$$\beta_{2i} = \gamma_{70} + \beta_{1i} + \gamma_{71} Group_i + \gamma_{72} Ham_{2i} + u_{7i}$$
$$Ham_{2i} = \gamma_{80} + Ham_{i1} + \gamma_{81} Group_i + u_{8i}$$

Group has a direct effect on the random effects (i.e., μ_{2i}, ϕ_{2i}, and β_{2i}):

- γ_{51}
- γ_{61}
- γ_{71}

Group also has an indirect effect through Ham_{2i}:

- on μ_{2i}: $\gamma_{81} \times \gamma_{52}$
- on ϕ_{2i}: $\gamma_{81} \times \gamma_{62}$
- on β_{2i}: $\gamma_{81} \times \gamma_{72}$

Mplus input

```
MODEL:
    %WITHIN%
    phi1 | na_pre ON na_pre&1;
    beta1 | na_pre ON UnPl_pre;
    phi2 | na_post ON na_post&1;
    beta2 | na_post ON UnPl_post;

    na_pre-UnPl_post WITH na_post-UnPl_post@0;
    UnPl_pre; UnPl_post;

    %BETWEEN%
    ham_pre ON Group;
    na_pre phi1 beta1 ON Group ham_pre;
    na_post ON na_pre@1 Group ham_post (e1-e3);
    phi2 ON phi1@1 Group ham_post (d1-d3);
    beta2 ON beta1@1 Group ham_post (b1-b3);
    ham_post ON ham_pre@1 Group (a1-a2);

model constraint:
    new (ind_GDm); ind_GDm=a2*e3; !indirect effect from group on change in mu
    new (ind_GDp); ind_GDp=a2*d3; !indirect effect from group on change in phi
    new (ind_GDb); ind_GDb=a2*b3; !indirect effect from group on change in beta
```

Mplus output

```
Between Level

 PHI2       ON
    PHI1               1.000      0.000      0.000      1.000      1.000

 BETA2      ON
    BETA1              1.000      0.000      0.000      1.000      1.000

 PHI1       ON
    GROUP              0.047      0.045      0.155     -0.043      0.135
    HAM_PRE            0.184      0.106      0.042     -0.024      0.387

 BETA1      ON
    GROUP              0.002      0.018      0.461     -0.033      0.039
    HAM_PRE           -0.104      0.044      0.007     -0.190     -0.019      *

 PHI2       ON
    GROUP             -0.050      0.065      0.212     -0.177      0.076
    HAM_POST           0.273      0.126      0.012      0.029      0.536      *

 BETA2      ON
    GROUP             -0.015      0.026      0.281     -0.069      0.034
    HAM_POST           0.020      0.049      0.340     -0.078      0.115

 HAM_PRE    ON
    GROUP              0.028      0.040      0.255     -0.054      0.102

 NA_PRE     ON
    GROUP             -0.098      0.125      0.204     -0.361      0.144
    HAM_PRE            1.334      0.287      0.000      0.789      1.904      *

 NA_POST    ON
    NA_PRE             1.000      0.000      0.000      1.000      1.000
    GROUP             -0.180      0.102      0.043     -0.384      0.024
    HAM_POST           0.641      0.197      0.001      0.256      1.039      *

 HAM_POST   ON
    HAM_PRE            1.000      0.000      0.000      1.000      1.000
    GROUP             -0.141      0.049      0.002     -0.237     -0.043      *
```

Mplus output

```
 Intercepts
    HAM_PRE            0.592      0.028      0.000       0.538       0.647      *
    HAM_POST          -0.049      0.033      0.075      -0.114       0.015
    NA_PRE             1.208      0.190      0.000       0.849       1.596      *
    NA_POST           -0.359      0.124      0.002      -0.604      -0.100      *
    PHI1               0.319      0.073      0.000       0.177       0.456      *
    BETA1             -0.061      0.029      0.020      -0.116      -0.005      *
    PHI2              -0.234      0.083      0.002      -0.401      -0.082      *
    BETA2             -0.004      0.032      0.466      -0.066       0.061

 Residual Variances
    HAM_PRE            0.046      0.006      0.000       0.035       0.061      *
    HAM_POST           0.067      0.009      0.000       0.052       0.089      *
    NA_PRE             0.380      0.057      0.000       0.290       0.507      *
    NA_POST            0.242      0.042      0.000       0.173       0.344      *
    PHI1               0.036      0.007      0.000       0.025       0.052      *
    BETA1              0.006      0.001      0.000       0.004       0.008      *
    PHI2               0.073      0.015      0.000       0.048       0.108      *
    BETA2              0.010      0.003      0.000       0.005       0.016      *

New/Additional Parameters
    IND_GDM           -0.086      0.044      0.004      -0.190      -0.019      *
    IND_GDP           -0.037      0.023      0.014      -0.091      -0.002      *
    IND_GDB           -0.002      0.007      0.341      -0.018       0.011
```

Considerations about the level 2 predictors...

We just did a model with:

Between level: Post-treatment phase

$$\mu_{2i} = \gamma_{50} + \mu_{1i} + \gamma_{51} Group_i + \gamma_{52} Ham_{2i} + u_{5i}$$
$$\phi_{2i} = \gamma_{60} + \phi_{1i} + \gamma_{61} Group_i + \gamma_{62} Ham_{2i} + u_{6i}$$
$$\beta_{2i} = \gamma_{70} + \beta_{1i} + \gamma_{71} Group_i + \gamma_{72} Ham_{2i} + u_{7i}$$
$$Ham_{2i} = \gamma_{80} + Ham_{i1} + \gamma_{81} Group_i + u_{8i}$$

Instead, we could use $\Delta Ham_i = Ham_{2i} - Ham_{1i}$, we get:

Between level: Post-treatment phase

$$\mu_{2i} = \gamma_{50} + \mu_{1i} + \gamma_{51} Group_i + \gamma_{52} \Delta Ham_i + u_{5i}$$
$$\phi_{2i} = \gamma_{60} + \phi_{1i} + \gamma_{61} Group_i + \gamma_{62} \Delta Ham_i + u_{6i}$$
$$\beta_{2i} = \gamma_{70} + \beta_{1i} + \gamma_{71} Group_i + \gamma_{72} \Delta Ham_i + u_{7i}$$
$$\Delta Ham_i = \gamma_{80} + \gamma_{81} Group_i + u_{8i}$$

Mplus input

```
DEFINE:  center UnPl_pre UnPl_post (groupmean);
         D_diff = ham_post - ham_pre;
         center ham_pre D_diff (grandmean);

ANALYSIS:   TYPE IS TWOLEVEL random; estimator=bayes;
            proc = 2; biter= (2000); bseed = 8179; thin = 10;

MODEL:
    %WITHIN%
    phi1 | na_pre ON na_pre&1;
    beta1 | na_pre ON UnPl_pre;
    phi2 | na_post ON na_post&1;
    beta2 | na_post ON UnPl_post;

    na_pre-UnPl_post WITH na_post-UnPl_post@0;
    UnPl_pre; UnPl_post;

    %BETWEEN%
    ham_pre ON Group;
    na_pre phi1 beta1 ON Group ham_pre;
    na_post ON na_pre@1 Group D_diff (e1-e3);
    phi2 ON phi1@1 Group D_diff (d1-d3);
    beta2 ON beta1@1 Group D_diff (b1-b3);
    D_diff ON Group (a2);

model constraint:
    new (ind_GDm); ind_GDm=a2*e3; !indirect effect from group on change in mu
    new (ind_GDp); ind_GDp=a2*d3; !indirect effect from group on change in phi
    new (ind_GDb); ind_GDb=a2*b3; !indirect effect from group on change in beta
```

Mplus output

Between Level

PHI2 ON						
PHI1	1.000	0.000	0.000	1.000	1.000	
BETA2 ON						
BETA1	1.000	0.000	0.000	1.000	1.000	
PHI1 ON						
GROUP	0.044	0.045	0.163	-0.045	0.132	
HAM_PRE	0.217	0.107	0.020	0.015	0.437	*
BETA1 ON						
GROUP	0.003	0.019	0.437	-0.032	0.041	
HAM_PRE	-0.109	0.044	0.007	-0.195	-0.022	*
PHI2 ON						
GROUP	-0.049	0.067	0.231	-0.180	0.083	
D_DIFF	0.197	0.130	0.068	-0.069	0.452	
BETA2 ON						
GROUP	-0.011	0.025	0.331	-0.058	0.037	
D_DIFF	0.014	0.045	0.370	-0.073	0.104	
HAM_PRE ON						
GROUP	0.026	0.041	0.270	-0.058	0.105	
NA_PRE ON						
GROUP	-0.104	0.121	0.199	-0.338	0.138	
HAM_PRE	1.408	0.282	0.000	0.827	1.948	*
NA_POST ON						
NA_PRE	1.000	0.000	0.000	1.000	1.000	
GROUP	-0.111	0.099	0.135	-0.299	0.084	
D_DIFF	1.050	0.183	0.000	0.685	1.398	*
D_DIFF ON						
GROUP	-0.145	0.048	0.002	-0.238	-0.051	*

Mplus output

```
 Intercepts
    HAM_PRE           -0.012       0.028      0.333      -0.069       0.045
    D_DIFF             0.070       0.034      0.020       0.003       0.137      *
    NA_PRE             2.026       0.082      0.000       1.865       2.178      *
    NA_POST           -0.081       0.065      0.105      -0.214       0.042
    PHI1               0.426       0.032      0.000       0.362       0.486      *
    BETA1             -0.125       0.013      0.000      -0.151      -0.100      *
    PHI2              -0.095       0.047      0.023      -0.191      -0.003      *
    BETA2              0.002       0.017      0.461      -0.032       0.034

 Residual Variances
    HAM_PRE            0.046       0.006      0.000       0.036       0.060      *
    D_DIFF             0.067       0.009      0.000       0.051       0.089      *
    NA_PRE             0.377       0.056      0.000       0.283       0.503      *
    NA_POST            0.196       0.035      0.000       0.140       0.274      *
    PHI1               0.037       0.007      0.000       0.025       0.054      *
    BETA1              0.006       0.001      0.000       0.004       0.009      *
    PHI2               0.077       0.015      0.000       0.051       0.111      *
    BETA2              0.008       0.002      0.000       0.004       0.014      *

New/Additional Parameters
    IND_GDM           -0.148       0.058      0.002      -0.280      -0.049      *
    IND_GDP           -0.026       0.022      0.070      -0.079       0.009
    IND_GDB           -0.002       0.007      0.370      -0.017       0.012
```

Outline

- Modeling the dynamics of ILD
- Separating between-person and within-person variance
- Application 1: Daily negative affect and depressive symptomatology
- Application 2: Intervention study with ESM
- **Application 3: Dyadic daily diary data**
- Application 4: Latent AR(1) model
- Discussion

General and relationship specific affect

Ferrer gathered daily diary data from couples regarding their:

- general positive affect that day (G-PA)
- general negative affect that day (G-NA)
- relationship specific positive affect that day (RS-PA)
- relationship specific negative affect that day (RS-NA)

Hence, for each of the 193 dyads there are 8 variables.

They were measured on 52-108 days.

Mplus summary of the data

```
SUMMARY OF DATA

     Number of clusters                         193

      Size (s)     Cluster ID with Size s

        52         120 196
        53         132 129
        54         119
        55         105 123 155 156 189 193 126 215 216 220 229 230 127
                   314 315 325
        56         160 164 183 184 185 111 125 136 211 139 142 218 146
                   222 224 226 153 154 231 232 233 237 241 242 243 244
                   245 246 249 250 258 262 269 287 288 289 293 298 299
                   305 121 309 122 158 316 319 159
        57         173 221 179 252 257 180 225 264 266 267 268 104 284
                   285 286 227 141 157 292 186 295 102 109 300 302 303
                   304 234 135 238 110 212 169 318 170 321 172 327 331
                   334
        58         101 174 113 108 228 181 273
        59         223 107 106 114
        61         621 622
        62         517 118 150
        63         133
        65         255
        82         700
        85         654
        88         699
        89         560 511 701
        90         510 562 563 703 711
        91         571 552 554 505 694 697 501 528 565 567 693 569
        92         609 615 530 540 648 650 504 667 673 692 600 524 556
                   503 573 577 579 709 604 714
        93         605 608 586 661 590 508 675 707 683 687 514 655
        94         518 595 523
        95         502 557 576 587
        96         678
        97         660 658
        98         712
       108         651
```

Multilevel VAR(1)

Within level: Vector autoregressive model

$$\begin{bmatrix} GPAM^*_{it} \\ GNAM^*_{it} \\ RSPAM^*_{it} \\ RSNAM^*_{it} \\ GPAF^*_{it} \\ GNAF^*_{it} \\ RSPAF^*_{it} \\ RSNAF^*_{it} \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} & \phi_{16} & \phi_{17} & \phi_{18} \\ \phi_{21} & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} & \phi_{26} & \phi_{27} & \phi_{28} \\ \phi_{31} & \phi_{32} & \phi_{33} & \phi_{34} & \phi_{35} & \phi_{36} & \phi_{37} & \phi_{38} \\ \phi_{41} & \phi_{42} & \phi_{43} & \phi_{44} & \phi_{45} & \phi_{46} & \phi_{47} & \phi_{48} \\ \phi_{51} & \phi_{52} & \phi_{53} & \phi_{54} & \phi_{55} & \phi_{56} & \phi_{57} & \phi_{58} \\ \phi_{61} & \phi_{62} & \phi_{63} & \phi_{64} & \phi_{65} & \phi_{66} & \phi_{67} & \phi_{68} \\ \phi_{71} & \phi_{72} & \phi_{73} & \phi_{74} & \phi_{75} & \phi_{76} & \phi_{77} & \phi_{78} \\ \phi_{81} & \phi_{82} & \phi_{73} & \phi_{84} & \phi_{85} & \phi_{86} & \phi_{87} & \phi_{88} \end{bmatrix} \begin{bmatrix} GPAM^*_{it-1} \\ GNAM^*_{it-1} \\ RSPAM^*_{it-1} \\ RSNAM^*_{it-1} \\ GPAF^*_{it-1} \\ GNAF^*_{it-1} \\ RSPAF^*_{it-1} \\ RSNAF^*_{it-1} \end{bmatrix} + \begin{bmatrix} \zeta_{1it} \\ \zeta_{2it} \\ \zeta_{3it} \\ \zeta_{4it} \\ \zeta_{5it} \\ \zeta_{6it} \\ \zeta_{7it} \\ \zeta_{8it} \end{bmatrix}$$

which gives:

$GPAM^*_{it} = \phi_{11} GPAM^*_{it-1} + \phi_{12} GNAM^*_{it-1} + \phi_{13} RSPAM^*_{it-1} + \phi_{14} RSNAM^*_{it-1} + \phi_{15} GPAF^*_{it-1} + \phi_{16} GNAF^*_{it-1} + \phi_{17} RSPAF^*_{it-1} + \phi_{18} RSNAF^*_{it-1} + \zeta_{1it}$

etc.

Multilevel VAR(1)

Within level: Residual covariance matrix

$$\begin{bmatrix} \zeta_{1it} \\ \zeta_{2it} \\ \dots \\ \zeta_{8it} \end{bmatrix} \sim MN(\mathbf{0}, \mathbf{\Theta}^*)$$

Hence, we estimate $8 \times 8 = 64$ lagged parameters, and $8 \times 9/2 = 36$ variances and covariances at the within-person level.

Between level: Fixed and random effects

$$\begin{bmatrix} \mu_{1i} \\ \mu_{2i} \\ \dots \\ \mu_{8i} \end{bmatrix} \sim MN(\boldsymbol{\gamma}, \mathbf{\Psi})$$

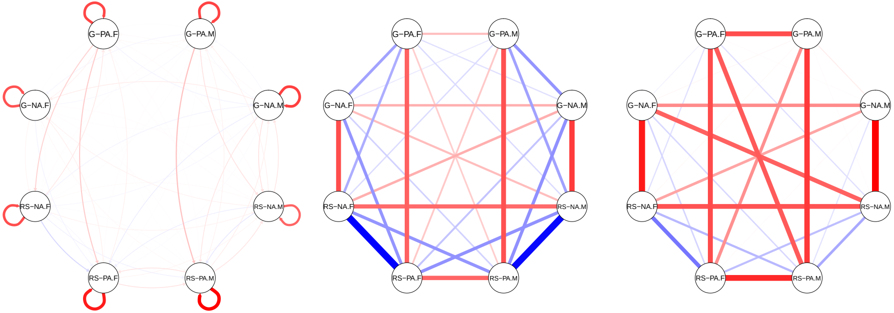
Hence, we estimate 8 grand means, and $8 \times 9/2 = 36$ variances and covariances at the between-person level. In total: 144 parameters.

Mplus input for multilevel VAR(1)

```
VARIABLE:
 names =   dyad day
           GPAM GNAM RSPAM RSNAM
           GPAF GNAF RSPAF RSNAF
           RelSat1M RelSat1F
           RelSat2M RelSat2F
           BrUpM BrUpF;
 usevar =  GPAM-RSNAF;
 lagged =  GPAM-RSNAF(1);
 cluster = dyad;
 missing = all(999);

ANALYSIS:  TYPE IS TWOLEVEL; estimator = bayes;
           proc = 2; biter = (5000); bseed = 574;

MODEL:
   %WITHIN%
   GPAM-RSNAF ON GPAM&1-RSNAF&1;
   GPAM-RSNAF WITH GPAM-RSNAF;

   %BETWEEN%
   GPAM-RSNAF WITH GPAM-RSNAF;
```

Three networks

Lagged, within-person (residual), and between-person networks:



Note:

- the lagged network is based on the within-person standardized lagged relationships
- the within-person residual network is based on within-person correlated residuals
- the between-person network is based on the correlated within-person means

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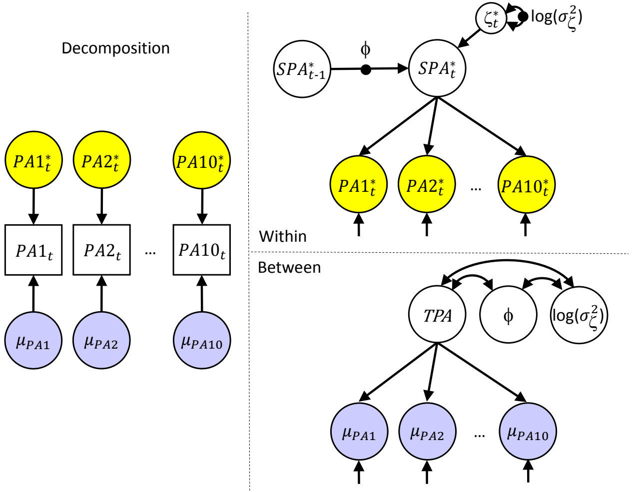
- Separating between-person and within-person variance
- Application 1: Daily negative affect and depressive symptomatology
- Application 2: Intervention study with ESM
- Application 3: Dyadic daily diary data
- **Application 4: Latent AR(1) model**
- Discussion

Multilevel AR factor model

Using the 10 indicators of PA from the COGITO study, we can specify a multilevel factor model:



Multilevel latent AR(1) model

Decomposition

$$\mathbf{y}_{it} = \boldsymbol{\mu}_i + \mathbf{y}^*_{it}$$

Within level: State positive affect

$$\mathbf{y}^*_{it} = \boldsymbol{\Lambda}^* SPA^*_{it} + \boldsymbol{\epsilon}^*_i \qquad \boldsymbol{\epsilon}^*_i \sim MN(\mathbf{0}, \boldsymbol{\Theta})$$

$$SPA^*_{it} = \phi_i SPA^*_{i,t-1} + \zeta^*_{it} \qquad \zeta^*_{it} \sim N(0, \sigma^2_{\zeta,i})$$

Between level: Trait positive affect

$$\boldsymbol{\mu}_i = \nu + \boldsymbol{\Lambda} TPA_i + \boldsymbol{\epsilon}_i$$

$$\begin{bmatrix} TPA_i \\ \phi_i \\ log(\sigma^2_{\zeta,i}) \end{bmatrix} = \begin{bmatrix} \gamma_{TPA} \\ \gamma_{\phi} \\ \gamma_{logVar} \end{bmatrix} + \begin{bmatrix} u_{TPA,i} \\ u_{\phi,i} \\ u_{logVar,i} \end{bmatrix}$$

Mplus input latent AR(1) model

```
VARIABLE:
names       = ID sessdate na1 na2 na3 na4 na5 na6 na7 na8 na9 na10
                pa1 pa2 pa3 pa4 pa5 pa6 pa7 pa8 pa9 pa10 sessionNr
                age_pre sex CESDpre CESDpost dayNA dayPA older;
cluster     =   ID;
usevar      =   pa1-pa10 sessdate;
tinterval   =   sessdate(1);
missing     =   all(-999);

ANALYSIS:   TYPE IS TWOLEVEL RANDOM; estimator=bayes;
            proc = 2; biter = (5000); bseed = 297; thin = 10;

MODEL:
    %WITHIN%
    SPA BY pa1-pa10 (&1);         ! FACTOR MODEL WITHIN
    SPA BY pa2-pa10 (LW2-LW10);  ! GIVE LABELS
    phi | SPA ON SPA&1;           ! LATENT AR(1)
    logVZ | SPA;                  ! RANDOM INN VAR

    %BETWEEN%
    TPA BY pa1-pa10 (LB1-LB10); ! FACTOR MODEL BETWEEN
    TPA WITH phi; TPA phi WITH logVZ;

    model constraint:             ! COMPARE FACTOR LOADINGS
    new (difL2); difL2=LB2-LW2;
    new (difL3); difL3=LB3-LW3;
    new (difL4); difL4=LB4-LW4;
    new (difL5); difL5=LB5-LW5;
    new (difL6); difL6=LB6-LW6;
    new (difL7); difL7=LB7-LW7;
    new (difL8); difL8=LB8-LW8;
    new (difL9); difL9=LB9-LW9;
    new (difL10); difL10=LB10-LW10;

OUTPUT: TECH1 TECH8 STDYX;
```

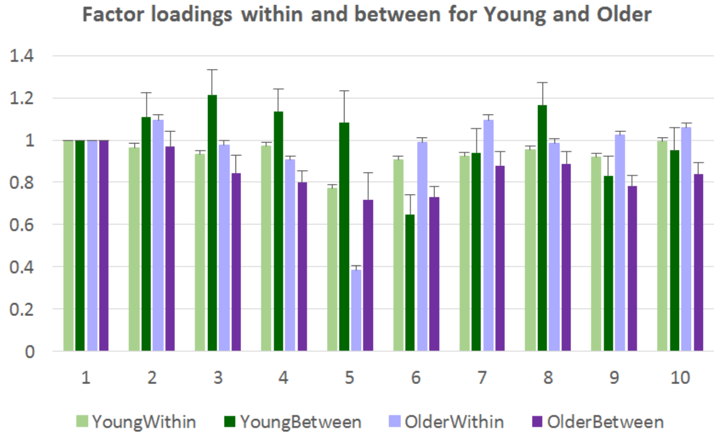
Mplus output: Comparing factor loadings across levels

New/Additional Parameters

DIFL2	-0.106	0.076	0.090	-0.242	0.060	
DIFL3	-0.118	0.089	0.101	-0.277	0.069	
DIFL4	-0.095	0.060	0.077	-0.199	0.037	
DIFL5	0.361	0.129	0.002	0.117	0.621	*
DIFL6	-0.246	0.057	0.001	-0.346	-0.121	*
DIFL7	-0.202	0.076	0.009	-0.334	-0.037	*
DIFL8	-0.080	0.061	0.107	-0.187	0.053	
DIFL9	-0.223	0.054	0.000	-0.315	-0.101	*
DIFL10	-0.199	0.060	0.003	-0.305	-0.066	*

Conclusion: 5 out of 10 factor loadings show evidence for being different across levels.

Factor loadings within-between for young-older



PA5 is the item "stolz"

Other items: 1) enthusiastic; 2) excited; 3) strong; 4) interested; 5) proud; 6) alert; 7) inspired; 8) determined; 9) attentive; 10) active

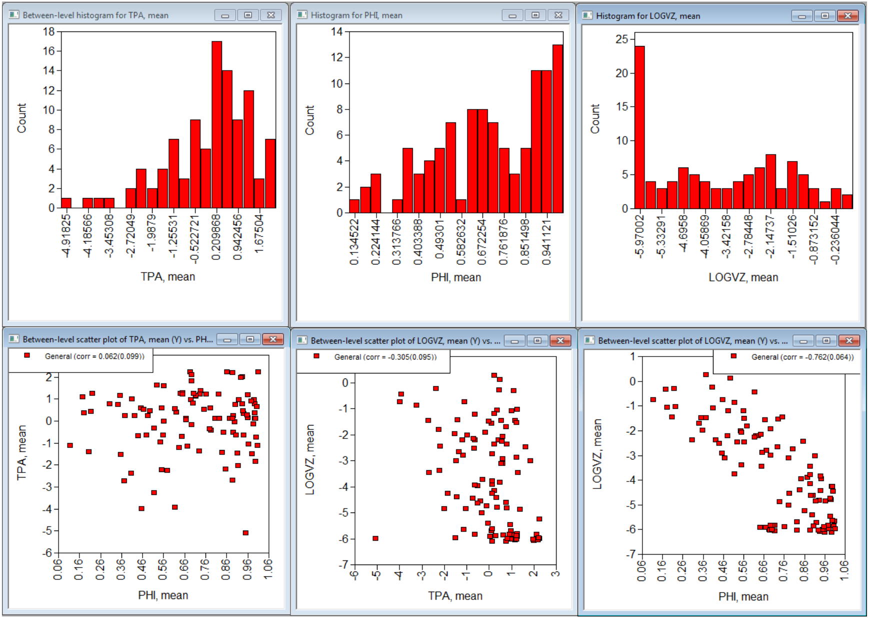
Mplus output: R-square

```
R-SQUARE

Within-Level R-Square Averaged Across Clusters

                                Posterior  One-Tailed         95% C.I.
    Variable        Estimate       S.D.      P-Value   Lower 2.5%  Upper 2.5%

    PA1                0.291       0.009      0.000       0.273       0.310
    PA2                0.314       0.010      0.000       0.293       0.333
    PA3                0.252       0.010      0.000       0.233       0.272
    PA4                0.302       0.010      0.000       0.282       0.323
    PA5                0.057       0.007      0.000       0.045       0.071
    PA6                0.305       0.010      0.000       0.285       0.325
    PA7                0.260       0.010      0.000       0.241       0.282
    PA8                0.273       0.010      0.000       0.254       0.294
    PA9                0.366       0.010      0.000       0.346       0.386
    PA10               0.339       0.010      0.000       0.319       0.360
[...]
    SPA                0.549       0.012      0.000       0.525       0.573

Between Level
[...]
    PA1                0.767       0.045      0.000       0.664       0.843
    PA2                0.844       0.031      0.000       0.775       0.895
    PA3                0.614       0.064      0.000       0.474       0.728
    PA4                0.876       0.025      0.000       0.819       0.916
    PA5                0.295       0.077      0.000       0.149       0.450
    PA6                0.872       0.027      0.000       0.811       0.914
    PA7                0.835       0.033      0.000       0.757       0.889
    PA8                0.947       0.013      0.000       0.917       0.966
    PA9                0.975       0.008      0.000       0.957       0.986
    PA10               0.935       0.015      0.000       0.900       0.958
```

Mplus output: Correlations at between level

```
STDYX Standardization

                                 Posterior  One-Tailed         95% C.I.
                     Estimate       S.D.      P-Value   Lower 2.5%  Upper 2.5%  Significance

Between Level
[...]

 TPA      WITH
    PHI                 0.067       0.110      0.263      -0.146       0.285
    LOGVZ              -0.303       0.096      0.002      -0.473      -0.100      *

 PHI      WITH
    LOGVZ              -0.728       0.063      0.000      -0.828      -0.584      *
```

Mplus output: Between-level plots

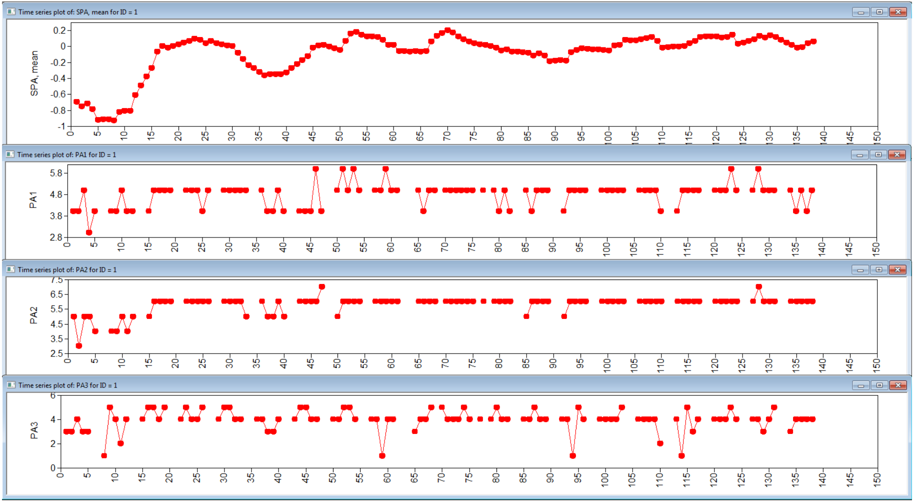


Mplus output: Estimated factor scores for ϕ_i

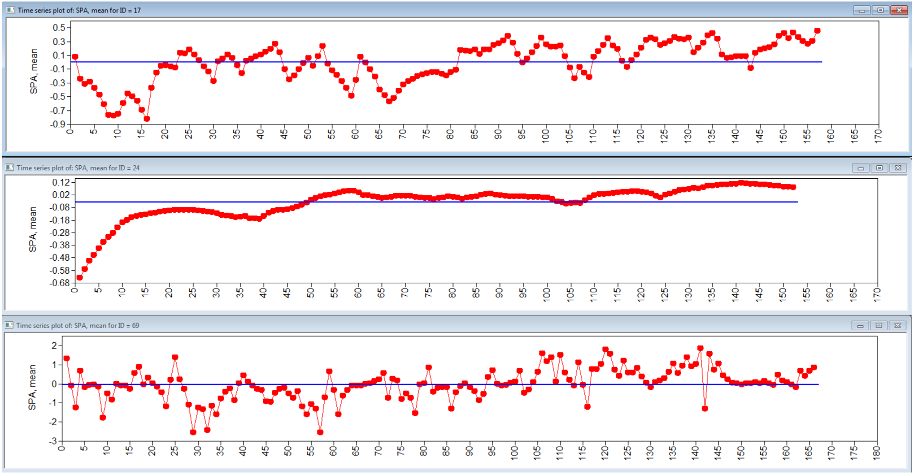
Using the statement:
OUTPUT: TECH1 TECH8 STDYX FSCOMPARISON;
PLOT: TYPE = PLOT3; FACTOR = ALL(1000);

```
Results for Factor PHI

Ranking  Cluster   Factor Score     Ranking  Cluster   Factor Score     Ranking  Cluster   Factor Score
    1      144       1.000              2       99       0.999              3      193       0.996
    4      156       0.994              5      132       0.989              6      151       0.989
    7      166       0.988              8      181       0.985              9       90       0.981
   10       53       0.979             11       87       0.969             12      112       0.968
   13      168       0.966             14       39       0.965             15        6       0.958
   16      157       0.949             17       94       0.942             18       58       0.941
   19      190       0.938             20      171       0.936             21        9       0.931
   22      142       0.926             23      163       0.924             24        1       0.904
   25      113       0.903             26      198       0.903             27       57       0.896
   28      170       0.894             29       92       0.890             30       24       0.886
   31       66       0.885             32       65       0.882             33      118       0.878
   34      108       0.877             35       40       0.874             36       59       0.839
   37      150       0.839             38       33       0.838             39       96       0.823
   40      199       0.820             41       47       0.813             42       54       0.808
   43       37       0.802             44       17       0.790             45       51       0.776
   46      133       0.775             47      200       0.755             48      127       0.739
   49       78       0.738             50       74       0.729             51      195       0.725
   52      203       0.722             53      146       0.719             54       97       0.713
   55       61       0.705             56      184       0.705             57       38       0.700
```

Estimated factor scores for SPA and observed scores



Estimated factor scores for 3 individuals



Multilevel latent AR(2) model

We can specify a multilevel autoregressive model of second order:

Decomposition

$$\mathbf{y}_{it} = \boldsymbol{\mu}_i + \mathbf{y}^*_{it}$$

Within level:

$$\mathbf{y}^*_{it} = \boldsymbol{\Lambda}^* PA^*_{it} + \boldsymbol{\epsilon}^*_i$$

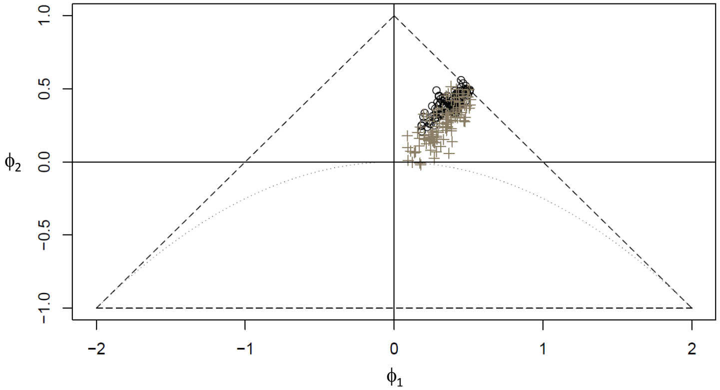
$$PA^*_{it} = \phi_{1i} PA^*_{i,t-1} + \phi_{2i} PA^*_{i,t-2} + \zeta^*_{it}$$

Between level:

$$\boldsymbol{\mu}_i = \nu + \boldsymbol{\Lambda} PA_i + \boldsymbol{\epsilon}_i$$

$$\begin{bmatrix} \eta_i \\ \phi_{1i} \\ \phi_{2i} \\ log(\sigma^2_\zeta) \end{bmatrix} = \begin{bmatrix} \gamma_\eta \\ \gamma_{\phi 1} \\ \gamma_{\phi 2} \\ \gamma_{logVar} \end{bmatrix} + \begin{bmatrix} u_{\eta,i} \\ u_{\phi 1,i} \\ u_{\phi 2,i} \\ u_{logVar,i} \end{bmatrix}$$

Autoregressive parameters



How about modeling a linear trend?

If we include **time as a within level predictor** in a multilevel AR model, we can do this in two ways:

Within level with time: Time has indirect effects

$$PA^*_{it} = \alpha_i time_{it} + \phi_{1i} PA^*_{i,t-1} + \zeta^*_{it}$$

where α_i is hard to interpret.

Within level with time: Trend with AR(1) residuals

$$PA^*_{it} = \beta_i time_{it} + a^*_{it}$$
$$a^*_{it} = \phi_{1i} a^*_{i,t-1} + \zeta^*_{it}$$

where β_i is the slope of the linear trend in the process.

The two specifications are related (see Hamaker, 2005):

- ϕ_i will be (almost) identical
- $\beta_i = \frac{\alpha_i}{1-\phi_i}$

Outline

- Modeling the dynamics of ILD
- Separating between-person and within-person variance
- Application 1: Daily negative affect and depressive symptomatology
- Application 2: Intervention study with ESM
- Application 3: Dyadic daily diary data
- Application 4: Latent AR(1) model
- **Discussion**

There is more...

DSEM in Mplus also allows for **cross-classified models**: observations are nested **in persons** AND **in occasions**.

Hence, you can have:

- **mean for each person** (μ_i, average score over time); these means have a distribution at the between-person level
- **mean for each time point** (μ_t, average score across people); these means have a distribution at the between-occasion level

You can also have:

- **person-specific regression coefficients** (e.g., β_i), that have distributions at the between-person level
- **time-specific regression coefficients** (e.g., β_t), that have distributions at the between-occasion level

Input of a cross-classified model

```
VARIABLE:
 names =   dyad day
 GPAM GNAM RSPAM RSNAM GPAF GNAF RSPAF RSNAF
 RelSat1M RelSat1F RelSat2M RelSat2F BrUpM BrUpF;
 usevar =  GPAM;
 lagged =  GPAM(1);
 cluster = dyad day;
 missing = all(999);

ANALYSIS:  TYPE IS CROSS RANDOM;
           estimator = bayes; proc = 2;
           biter = (3000); bseed = 1574;

MODEL:
   %WITHIN%
   phi | GPAM ON GPAM&1;

   %BETWEEN dyad%
   phi WITH GPAM;

   %BETWEEN day%
   GPAM;
   phi@0;
```

Cross-classified models

This approach is useful when:

- time is **meaningful** (e.g., days since quite smoking; trial since the beginning)
- you **expect a trend** (in mean or in regression coefficient), which may be in the same direction for most participants

Using the cross-classified part allows you to **explore the shape of the trend over time**.

Can be thought of as an **alternative** to the **TVEM** (time varying effect modeling) and **TVAR** (time varying autoregressive modeling).

But it requires:

- **longer time series** (especially for random autoregressions; e.g., $T>200$)
- **observations from multiple individuals per time point**

And more...

```
TITLE:     this is an example of a two-level time
           series analysis with a first-order
           autoregressive AR(1) IRT model for binary
           factor indicators with random thresholds,
           a random AR(1) slope, and a random
           residual variance
DATA:      FILE = ex9.35part2.dat;
VARIABLE:  NAMES = u1-u4 subject;
           CATEGORICAL = u1-u4;
           CLUSTER = subject;
ANALYSIS:  TYPE = TWOLEVEL RANDOM;
           ESTIMATOR = BAYES;
           PROCESSORS = 2;
           BITERATIONS = (3000);
MODEL:     %WITHIN%
           f BY u1-u4*(&1 1-4);
           s | f ON f&1;
           logvf | f;
           %BETWEEN%
           fb BY u1-u4* (1-4);
           [logvf@0];
           fb s logvf WITH fb s logvf;
OUTPUT:    TECH1 TECH8;
```

And there will be more...

Mplus v8.1 (or v8.2?) will also allow for N=1 and multilevel regime-switching models.

Features of **N=1 regime-switching models** (see Kim and Nelson, 1990):

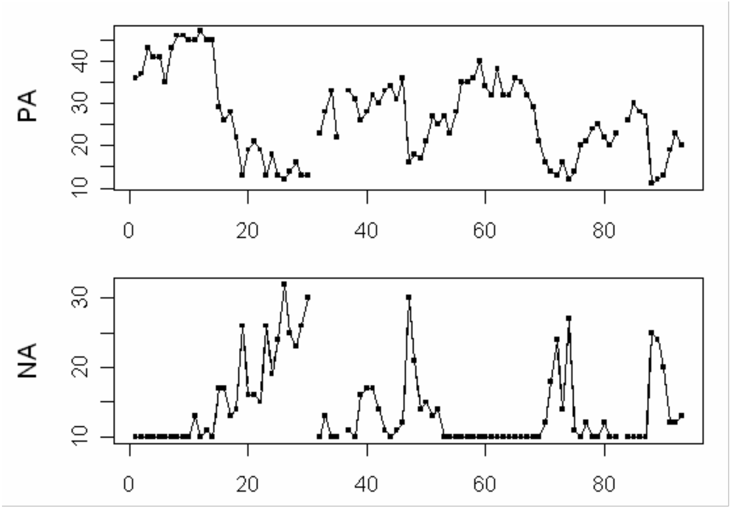
- two or more discrete states (or regimes)
- switching between these states is a hidden Markov process
- each state is characterized by its own process: different means, autoregression, cross-lagged regressions, etc.

Features of **multilevel regime-switching models**:

- switching probabilities can be random across individuals
- state-specific parameters can be random across individuals

Example: Bipolar disorder

Bipolar disorder is characterized by severe changes in affect and activity: Bipolar patients suffer from **manic** and **depressed episodes**.



Discussion: Model evaluation

Model fit and model comparison are unresolved issues at this point.

Model fit: Should we focus on explained variance, covariance, or lagged structure?

Model comparison:

- DIC is highly unreliable (check using different seeds!)
- DIC is not always comparable (see Celeux et al.)
- Bayes factors don't go well with uninformative priors

Discussion

Venues for future research:

- **samples sizes** (both N and T) and number of parameters
- **trends**: to detrend or not to detrend?
- **distributions**: how normal is normal?
- **model comparison**
- **model fit**

References

- Asparouhov, Hamaker & Muthén (2017). Dynamic latent class analysis. *Structural Equation Modeling: A Multidisciplinary Journal, 24*, 257-269.
- Asparouhov, Hamaker & Muthén (in preparation). Dynamic structural equation models.
- Hamaker (2012). Why researchers should think "within-person": A paradigmatic rationale. In M. R. Mehl & T. S. Conner (Eds.). *Handbook of Research Methods for Studying Daily Life*, 43-61. New York, NY: Guilford Publications.
- Hamaker, Asparouhov, Brose, Schmiedek & Muthén (submitted). At the frontiers of modeling intensive longitudinal data: Dynamic structural equation models for the affective measurements from the COGITO study. *Multivariate Behavioral Research*.
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- Schuurman, Ferrer, de Boer-Sonnenschein & Hamaker (2016). How to compare cross-lagged associations in a multilevel autoregressive model. *Psychological Methods, 21*, 206-221.

References and suggested readings

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- Raudenbush S.W. & Bryk, A.S. (2002). *Hierarchical linear models: Applications and data analysis methods (Second Edition)*. Thousand Oaks, CA: Sage Publications.
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